AP Calculus
Formulas for related rates

1. Ladder and other triangle problems:
   Pythagorean theorem: $a^2 + b^2 = c^2$

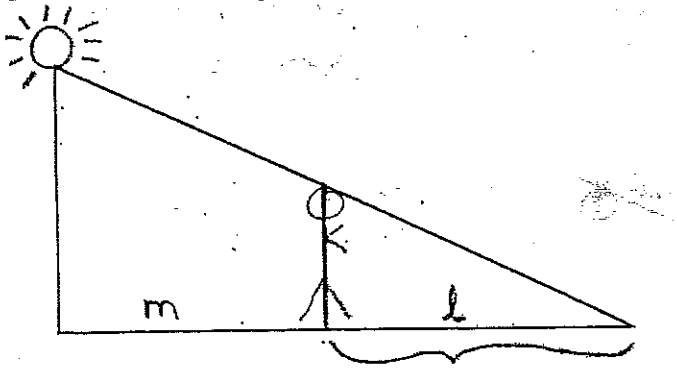
2. Sphere problems:
   $V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$

3. Circle problems:
   $A = \pi r^2$

4. Cone Problems:
   $V = \frac{1}{3}\pi r^2 h$

5. Shadow Problems:

$$\frac{\text{height of lamp post}}{\text{distance from lamp post to tip of shadow}} = \frac{\text{height of man}}{\text{distance from man to tip of shadow}}$$



$T = m + l$

$l = T - m$

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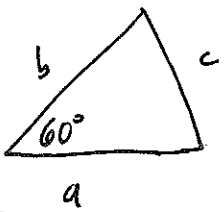

$c^2 = a^2 + b^2 - 2ab\cos 60°$ $\quad \cos 60° = \frac{1}{2}$, so $-2ab\cos 60 = -ab$

$2c \cdot \frac{dc}{dt} = 2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} - \frac{da}{dt} \cdot b - \frac{db}{dt} \cdot a$

At 4pm

$c^2 = 9^2 + 8^2 - 2(9)(8)\cos 60$

$c^2 = 73$

$c = \sqrt{73}$

$2\sqrt{73} \cdot \frac{dc}{dt} = 2(9)(3) + 2(8)(4) - (3)(8) - (4)(9)$

$2\sqrt{73} \cdot \frac{dc}{dt} = 58$

$\frac{dc}{dt} = \frac{58}{2\sqrt{73}}$

$\frac{dc}{dt} = \frac{29}{\sqrt{73}} \approx 3.39$ mph

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$V = \frac{1}{3}\pi r^2 h$, $r = 2h$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{1}{3}\pi \cdot 4h^3$

$V = \frac{4}{3}\pi h^3$

$\frac{dV}{dt} = 3 \cdot \frac{4}{3}\pi h^2$

$\frac{dV}{dt} = 4\pi h^2 \cdot \frac{dh}{dt}$

$20 = 4\pi (2)^2 \cdot \frac{dh}{dt}$

$20 = 16\pi \cdot \frac{dh}{dt}$

$\frac{20}{16\pi} = \frac{dh}{dt}$

$\frac{5}{4\pi} = \frac{dh}{dt} \approx .40 \frac{in}{sec}$

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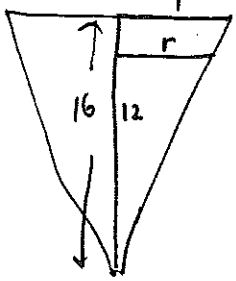

similar triangles

$\frac{16}{4} = \frac{h}{r}$

$16r = 4h$

$r = \frac{1}{4}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \cdot (\frac{1}{4}h)^2 \cdot h$

$V = \frac{1}{3}\pi \cdot \frac{1}{16}h^3$

$V = \frac{1}{48}\pi h^3$

$\frac{dV}{dt} = 3 \cdot \frac{1}{48}\pi h^2 \cdot \frac{dh}{dt}$

$-2 = \frac{1}{16}\pi \cdot (12)^2 \cdot \frac{dh}{dt}$

$-2 = 9\pi \cdot \frac{dh}{dt}$

$\frac{-2}{9\pi} = \frac{dh}{dt} \approx -.071 \frac{in}{sec}$

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(a) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$2 = 4\pi \cdot 3^2 \cdot \frac{dr}{dt}$

$2 = 36\pi \cdot \frac{dr}{dt}$

$\frac{1}{18\pi} = \frac{dr}{dt}$

$\frac{ft}{sec.}$

(b) $SA. = 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 8\pi \cdot 3 \cdot \frac{1}{18\pi}$

$\frac{dA}{dt} = \frac{4}{3}$ ft^2/sec