4) $\sqrt{2}\sin x - 1 = 0$

$\sin x = \frac{\sqrt{2}}{2}$

$x = 45°, 135°$

$x = \frac{\pi}{4}, \frac{3}{4}\pi$

$x = 45 + 360k$

$x = 135 + 360k$

$x = \frac{\pi}{4} + 2\pi k$

$x = \frac{3}{4}\pi + 2\pi k$

7) $\cos x = \pm\frac{1}{2}$

$x = 60°, 120°, 240°, 300°$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = 60 + 180k$

$x = 120 + 180k$

$x = \frac{\pi}{3} + \pi k$

$x = \frac{2}{3}\pi + \pi k$

OR

preferred

$x = \pm 60 + 180k$

$x = \pm\pi/3 + \pi k$

10) $\csc x = \pm 2$

$(\sin x = \pm\frac{1}{2})$

$x = 30, 150, 210, 330$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11}{6}\pi$

$x = 30 + 180k$

$x = 150 + 180k$

$x = \frac{\pi}{6} + \pi k$

$x = \frac{5\pi}{6} + \pi k$

OR

preferred

$\pm 30 + 180k$

$\pm\pi/6 + \pi k$

13) $\tan x = -\sqrt{3}$

$x = 120°, 300°$

$x = \frac{2}{3}\pi, \frac{5}{3}\pi$

$x = 120 + 180k$

$x = \frac{2}{3}\pi + \pi k$

$\cos x = -2$

No soln.

16) $\sin x(\tan x + 1) = 0$

$\sin x = 0$ $\quad$ $\tan x = -1$

$x = 0, 180, 360$

$x = 0, \pi, 2\pi$

$x = 180k$

$x = \pi k$

$x = 135°, 315$

$x = \frac{3}{4}\pi, \frac{7}{4}\pi$

$x = 135 + 180k$

$x = \frac{3}{4}\pi + \pi k$

18) $(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ $\quad$ $\sin x = 1$

$x = 210°, 330°$

$x = \frac{7}{6}\pi, \frac{11}{6}\pi$

$x = 210 + 360k$

$x = 330 + 360k$

$x = \frac{7}{6}\pi + \pi k$

$x = \frac{11}{6}\pi + \pi k$

$x = 90°$

$x = \pi/2$

$x = 90 + 360k$

$x = \frac{\pi}{2} + 2\pi k$

$x = 90, 210, 330$

$x = 90 + 120k$

$x = \frac{\pi}{2} + \frac{2}{3}\pi k$

21) $1-\cos^2 x = 4-2\cos^2 x$

$\cos^2 x = 3$

$\cos x = \pm\sqrt{3} \approx 1.73$

no solutions

23) $2\sin w + 1 = 0$

$\sin w = -\frac{1}{2}$

$w = 210°, 330°$

$3x = 210°, 330°$

$x = 70°, 110°, 190, 310, 230, 350$

$x = 70 + 120k$

$x = 110 + 120k$

29) $\cos w - 1 = 0$

$\cos w = 1$

$w = 0, 360$

$\frac{x}{2} = 0, 360$

$x = 0, 720$

$x = 0$

$x = 720k$

34) $3\tan^2 x(\tan x - 1) - 1(\tan x - 1) = 0$

$(\tan x - 1)(3\tan^2 x - 1) = 0$

$\tan x = 1$

$x = 45, 225$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = 45 + 180k$

$x = \frac{\pi}{4} + \pi k$

$\tan x = \pm\frac{\sqrt{3}}{3}$

$x = 30, 150, 210, 330$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = 30 + 180k$

$x = 150 + 180k$

$x = \frac{\pi}{6} + \pi k$

$x = \frac{5}{6}\pi + \pi k$

preferred

$x = \pm 30 + 180k$

$x = \pm\frac{\pi}{6} + 180k$

5) $\sin x = -\frac{\sqrt{3}}{2}$

$\begin{cases} x = 240, 300 \\ x = \frac{4}{3}\pi, \frac{5}{3}\pi \end{cases}$

$\begin{cases} x = 240 + 360k \\ x = 300 + 360k \\ x = \frac{4}{3}\pi + 2\pi k \\ x = \frac{5}{3}\pi + 2\pi k \end{cases}$

12) $\sec x = 0$

$\cos x =$ und.

no solns.

$\cos x = \frac{\sqrt{2}}{2}$

$\begin{cases} x = 45, 315 \\ x = \frac{\pi}{4}, \frac{7\pi}{4} \end{cases}$

$\begin{cases} x = 45 + 360k \\ x = 315 + 360k \\ x = \frac{\pi}{4} + 2\pi k \\ x = \frac{7\pi}{4} + 2\pi k \end{cases}$ OR $x = \pm 45 + 360k$ OR $x = \pm\frac{\pi}{4} + 2\pi k$

19) $\sin^2 x - 2\sin x - 3 = 0$

$(\sin x - 3)(\sin x + 1) = 0$

$\sin x = 3$ $\quad \sin x = -1$

no solns.

$\begin{cases} x = 270^\circ \\ x = \frac{3}{2}\pi \end{cases}$

$\begin{cases} x = 270 + 360k \\ x = \frac{3}{2}\pi + 2\pi k \end{cases}$

20) $3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0$ $\quad \tan x = \pm\frac{\sqrt{3}}{3}$

$\begin{cases} x = 0, 180, 360 \\ x = 0, \pi, 2\pi \end{cases}$

$\begin{cases} x = 180k \\ x = \pi k \end{cases}$

$x = 30, 150, 210, 330$

$x = 30 + 180k$

$x = 150 + 180k$

OR

$x = \pm 30 + 180k$

24) $2\cos w + 1 = 0$

$\cos w = -\frac{1}{2}$

$w = 120, 240$ $\longrightarrow$ $w = 120 + 360k$

$2x = 120, 240$ $\quad x = 60 + 180k$

$x = 60, 120$ $\quad w = 240 + 360k$

also $\quad x = 120 + 180k$

240, 300

31) $\tan\frac{x}{4} + \sqrt{3} = 0$

$\tan w = -\sqrt{3}$

$w = 120, 300 \rightarrow w = 120 + 180k$

$\frac{x}{4} = 120, 300 \quad x = 480 + 720k$

$x = 480, 1200$

no solns in $[0, 360]$

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40) $\csc x = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x = 60, 120, 240, 300$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = 60 + 180k \rightarrow x = \frac{\pi}{3} + \pi k$

$x = 120 + 180k \rightarrow x = \frac{2\pi}{3} + \pi k$

OR

$x = \pm 60 + 180k \rightarrow x = \pm\frac{\pi}{3} + \pi k$

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5) (35) $2\sin x(2\cos x + 1) - 1(2\cos x + 1) = 0$

$(2\sin x - 1)(2\cos x + 1) = 0$

$\sin x = \frac{1}{2}$ $\quad \cos x = -\frac{1}{2}$

$x = 30, 150$ $\quad \cos x = 120, 240$

$x = 30 + 360k$ $\quad x = 120 + 360k$

$x = 150 + 360k$ $\quad x = 240 + 360k$

$\frac{\pi}{6} + 360k$ $\quad \frac{2\pi}{3} + 2\pi k$

$\frac{5\pi}{6} + 360k$ $\quad \frac{4}{3}\pi + 2\pi k$

(44) $2(1 - \cos^2 x) - \cos x = 1$

$2 - 2\cos^2 x - \cos x = 1$

$0 = 2\cos^2 x + \cos x - 1$

$0 = (2\cos x - 1)(\cos x + 1)$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$

$x = 60, 300$ $\quad x = 180$

$x = 60 + 120k$

$x = \frac{\pi}{3} + \frac{2}{3}\pi k$