

(15) $3 \sec^2 x - 4 = 0$

$3 \sec^2 x = 4$

$\sec^2 x = \frac{4}{3}$

$\sec x = \pm \frac{2}{\sqrt{3}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = 30, 150, 210, 330$

$x = \pm 30 + 180k$

$x = \pm \frac{\pi}{6} + \pi k$

(23) $3 \tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$

$x = 30, 150, 210, 330$

$x = \pm 30 + 180k$

$x = \pm \frac{\pi}{6} + \pi k$

or $\tan^2 x - 3 = 0$

$\tan^2 x = 3$

$\tan x = \pm \sqrt{3}$

$x = 60, 120, 240, 300$

$x = \pm 60 + 180k$

$x = \pm \frac{\pi}{3} + \pi k$

(28) $2(1 - \cos^2 x) = 2 + \cos x$

$2 - 2 \cos^2 x = 2 + \cos x$

$-2 + 2 \cos^2 x \quad -2 + 2 \cos^2 x$

$0 = 2 \cos^2 x + \cos x$

$0 = \cos x (2 \cos x + 1)$

$\cos x = 0 \quad 2 \cos x + 1 = 0$

$x = 90, 270$

$x = 90 + 180k$

$x = \frac{\pi}{2} + \pi k$

$\cos x = -\frac{1}{2}$

$x = 120, 240$

$x = \pm 120 + 360k$

$x = \pm \frac{2\pi}{3} + 2\pi k$

(30) $\sec x \csc x - 2 \cos x = 0$

$\csc x (\sec x - 2) = 0$

$\csc x = 0$

~~no solutions~~

$\sin x = \frac{1}{0}$

no solutions

$\sec x - 2 = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

$x = 60, 300$

$x = \pm 60 + 360k$

$x = \pm \frac{\pi}{3} + 2\pi k$

(38) $(2 \sin x + 1)(\sin x + 1) = 0$

$2 \sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$x = 210, 330$

$x = 210 + 360k$

$x = 330 + 360k$

$x = \frac{7\pi}{6} + 2\pi k$

$x = \frac{11\pi}{6} + 2\pi k$

$\sin x + 1 = 0$

$\sin x = -1$

$x = 270$

$x = 270 + 360k$

$x = \frac{3}{2}\pi + 2\pi k$

(44) $2 \sin^2 x (2 \sin x + 1) - 1(2 \sin x + 1) = 0$

$(2 \sin x + 1)(2 \sin^2 x - 1) = 0$

$\sin x = -\frac{1}{2}$

$x = 210, 330$

$x = 210 + 360k$

$x = 330 + 360k$

$x = \frac{7}{6}\pi + 2\pi k$

$x = \frac{11}{6}\pi + 2\pi k$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{2}}$

$\sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

$x = 45, 135, 225, 315$

$x = 45 + 90k$

$x = \frac{\pi}{4} + \frac{\pi}{2} k$

11) $\cos x = -\frac{1}{2}$
 $x = 120, 240$
 $x = \pm 120 + 360k$
 $x = \pm \frac{2\pi}{3} + 2\pi k$

12) $\sin x = \frac{1}{2}$
 $x = 30, 150$
 $x = 30 + 360k$
 $x = 150 + 360k$
 $x = \frac{\pi}{6} + 2\pi k$
 $x = \frac{5\pi}{6} + 2\pi k$

13) $\csc x = \frac{2}{\sqrt{3}}$
 $\sin x = \frac{\sqrt{3}}{2}$
 $x = 60, 120$
 $x = 60 + 360k$
 $x = 120 + 360k$
 $x = \frac{\pi}{3} + 2\pi k$
 $x = \frac{2\pi}{3} + 2\pi k$

14) $\tan x = -1$
 $x = 135, 315$
 $x = 135 + 180k$
 $x = \frac{3\pi}{4} + \pi k$

16) $\csc x = \pm\sqrt{2}$
 $\sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$
 $x = 45, 135, 225, 315$
 $x = \pm 45 + 180k$
 $x = \pm \frac{\pi}{4} + \pi k$

19) $\sin x = \pm \frac{\sqrt{3}}{2}$
 $x = 60, 120, 240, 300$
 $x = \pm 60 + 180k$
 $x = \pm \frac{\pi}{3} + \pi k$

21) $1 - \cos^2 x = 3 \cos^2 x$
 $1 = 4 \cos^2 x$
 $\pm \frac{1}{2} = \cos x$
 $x = 60, 120, 240, 300$
 $x = \pm 60 + 180k$
 $x = \pm \frac{\pi}{3} + \pi k$

25) $\cos^2 x - \cos x = 0$
 $\cos x (\cos x - 1) = 0$
 $\cos x = 0$ $\cos x = 1$
 $x = 90, 270$ $x = 0, 180, 360$
 $x = 90k$
 $x = \frac{\pi}{2} k$

20) $\tan x = \pm 1$
 $x = 45, 135, 225, 315$
 $x = 45 + 90k$
 $x = \frac{\pi}{4} + \frac{\pi}{2} k$

27) $3 \tan^2 x - \tan x = 0$
 $\tan x (3 \tan^2 x - 1) = 0$
 $\tan x = 0$ $\tan^2 x = \frac{1}{3}$
 $x = 0, 180, 360$ $\tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$
 $x = 180k$ $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$
 $x = \pi k$ $x = \pm 30 + 180k$
 $x = \pm \frac{\pi}{6} + \pi k$

29) $\sec^2 x - \sec x - 2 = 0$
 $(\sec x - 2)(\sec x + 1) = 0$
 $\sec x = 2$ $\sec x = -1$
 $\cos x = \frac{1}{2}$ $\cos x = -1$
 $x = 60, 120, 240$ $x = 180$
 $x = \pm 120 + 360k$ $x = 180 + 360k$
 $x = \pm \frac{2\pi}{3} + 2\pi k$ $x = \pi + 2\pi k$

39) $2(1 + \tan^2 x) + \tan^2 x - 3 = 0$
 $3 \tan^2 x - 1 = 0$
 $\tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$

See #27