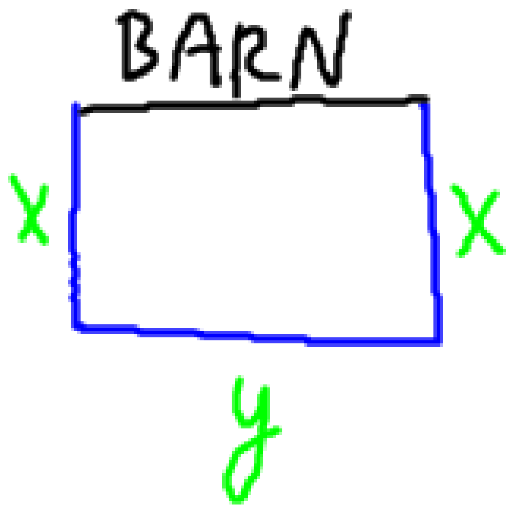
A Classic Problem

You have 40 feet of fence to enclose a rectangular garden along the side of a barn. What is the maximum area that you can enclose?



$$2x + y = 40$$

$$y = 40 - 2x$$

$$A = xy$$

$$A = x(40 - 2x)$$

$$A = 40x - 2x^2$$

$$A' = 40 - 4x$$

$$0 = 40 - 4x$$

$$4x = 40$$

$$x = 10$$

$$A = 10(20)$$

$$A = 200 \text{ ft}^2$$

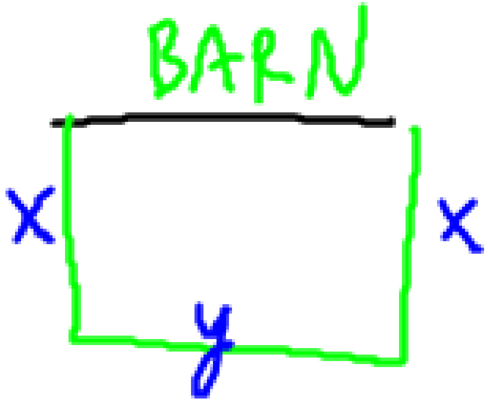
To find the maximum (or minimum) value of a function:

1. Write it in terms of one variable.
2. Find the first derivative and set it equal to zero.
3. Check the end points if necessary.

2

You have \$400 to fence in garden.
Fence along ends costs \$2 per foot.
Front fence costs \$3 per foot.
Find dimensions that will maximize area.

(2)



$A = xy$

$2x + 2x + 3y$

$4x + 3y = 400$

$x = \frac{400 - 3y}{4}$

$x = 100 - \frac{3}{4}y$

You have \$400 to fence in garden.
Fence along ends costs \$2 per foot.
Front fence costs \$3 per foot.
Find dimensions that will maximize area.

$$A = xy$$

$$A = \left(100 - \frac{3}{4}y\right)y$$

$$A = 100y - \frac{3}{4}y^2$$

$$A' = 100 - \frac{3}{2}y$$

$$0 = 100 - \frac{3}{2}y$$

$$\frac{3}{2}y = 100$$

$$y = \frac{200}{3}$$

$$A = \left(100 - \frac{3}{4} \cdot \frac{200}{3}\right)\frac{200}{3}$$

$$A = \frac{10000}{3}\ ft^2$$

$$A = 3333\frac{1}{3}\ ft^2$$

③ Currently you sell 20 tickets each day when the price is $15 per ticket. You think for every $2 drop in ticket price, you will sell 5 more tickets each day. What price will maximize ~~profit~~. revenue?

$R = (\text{price per})(\#\text{sold})$

$R = (15)(20)$

$x = \#$ of $2 drops

$R = (15 - 2x)(20 + 5x)$

$R = 300 + 35x - 10x^2$

$R' = 35 - 20x$

$0 = 35 - 20x$

$x = 1.75$

Price $= 15 - 2(1.75) = 11.50$

sold $= 20 + 5(1.75) = 28.75$

Revenue $= (11.50)(28.75) =$ \$330.60