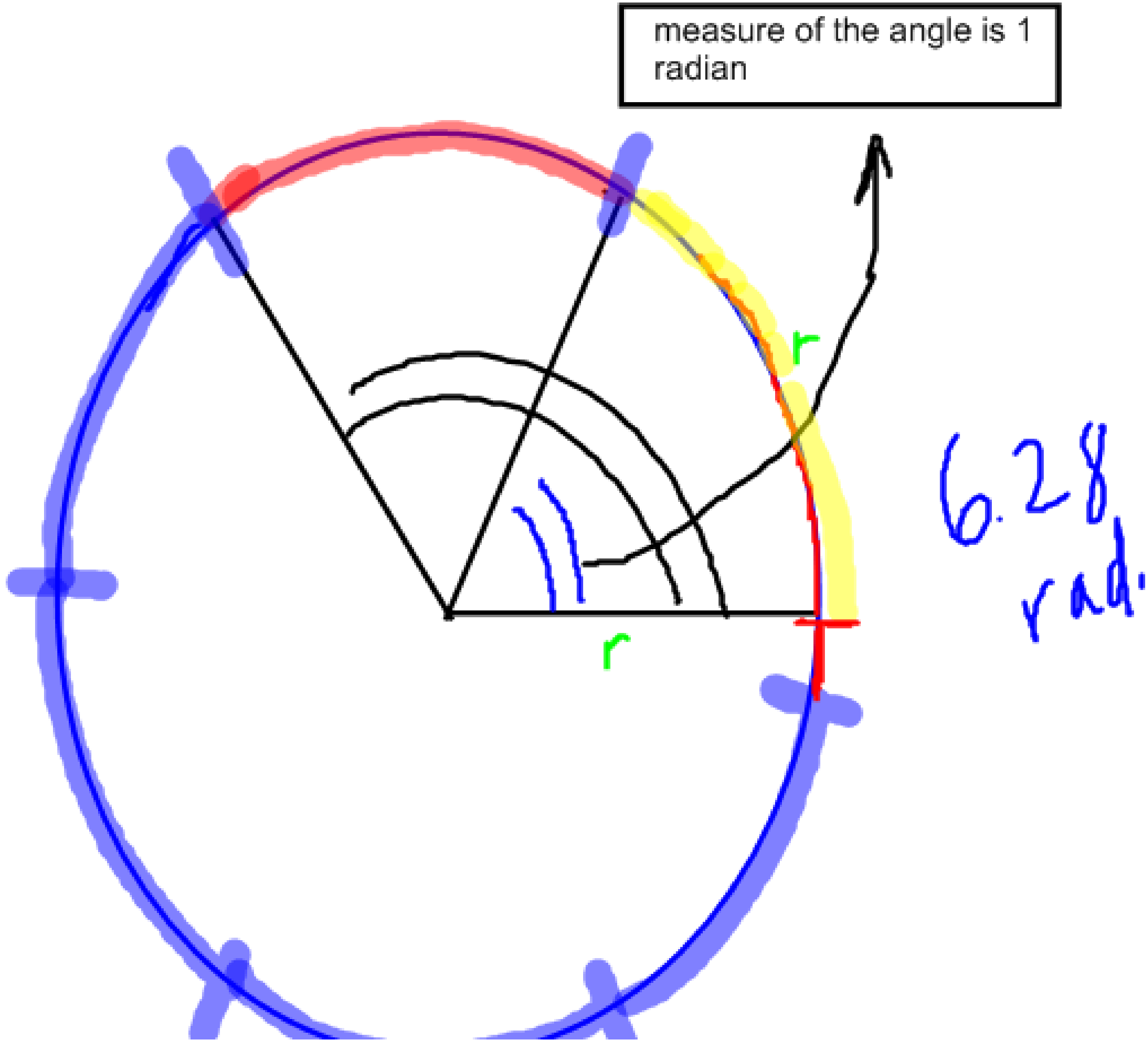
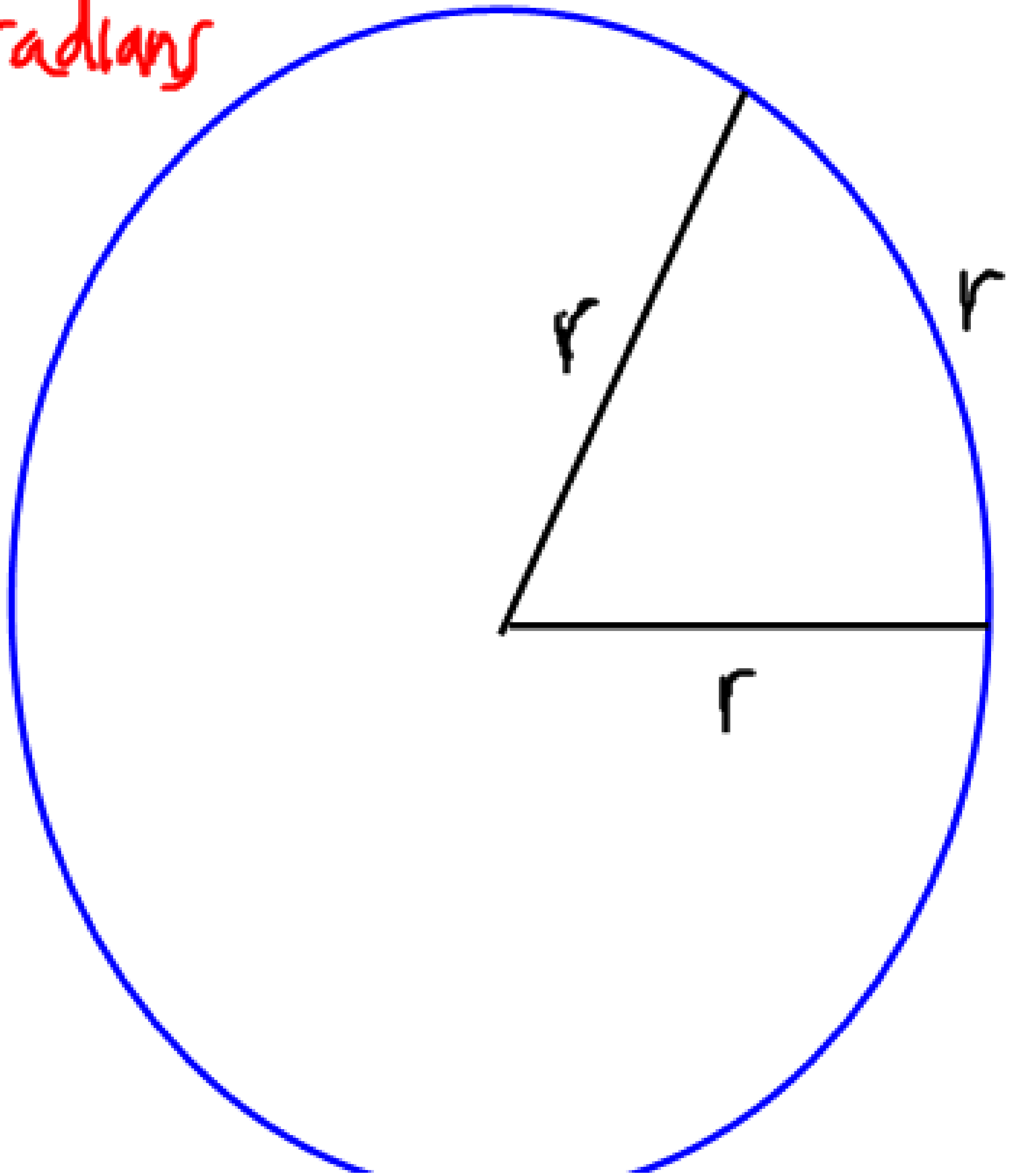

measure of the angle is 1 radian
r
r
6.28 rad.

$360^\circ = 2\pi$ radians

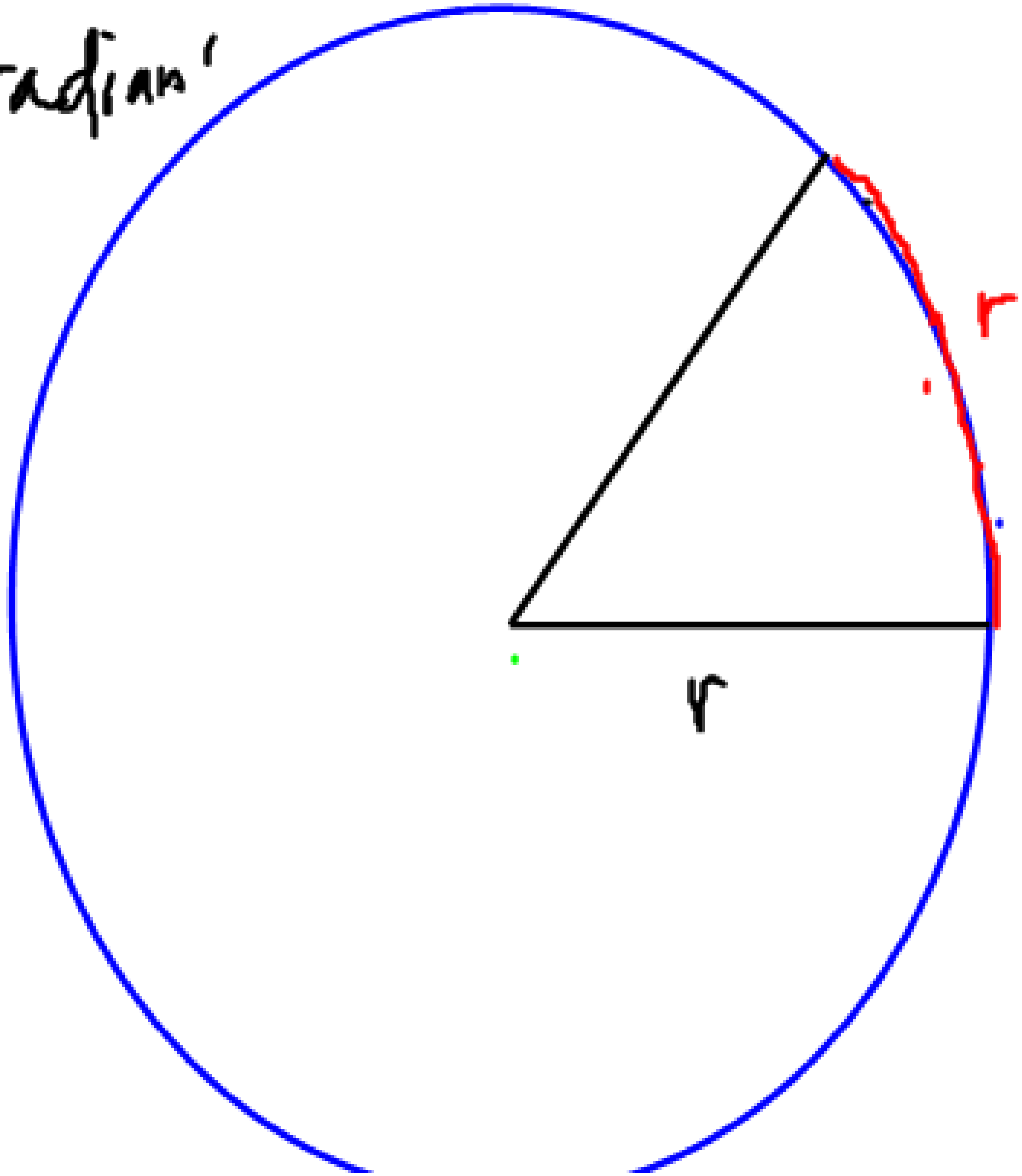
$180^\circ = \pi$ radians

$C = 2\pi r$



$360° = 2\pi$ radians

$180° = \pi$ radians



1. $40\cancel{^\circ} \cdot \dfrac{\pi \text{ rad}}{180\cancel{^\circ}} = \dfrac{40\pi}{180} \text{ radians} = \dfrac{2}{9}\pi \text{ radians}$

D $\rightarrow$ R

2. $210\cancel{^\circ} \cdot \dfrac{\pi \text{ rad}}{180\cancel{^\circ}} = \dfrac{7}{6}\pi \text{ radians}$

③ $\frac{2}{5}\pi \cancel{\text{rad}} \cdot \frac{180°}{\pi \cancel{\text{rad}}} = \frac{2(180)°}{5} = 72°$

④ $1 \cancel{\text{radian}} \cdot \frac{180°}{\pi \cancel{\text{radians}}} = \left(\frac{180}{\pi}\right)° \approx 57.3°$

⑤ $\frac{7}{4}\pi \text{ rad}$

$1^\circ = 60 \text{ minutes} = 60'$

$1 \text{ minute} = 60 \text{ seconds} = 60''$

$1^\circ = 3600 \text{ seconds}$

DMS $\rightarrow$ D

⑥ $37^\circ 24' 36'' = 37^\circ + \left(\frac{24}{60}\right)^\circ + \left(\frac{36}{3600}\right)^\circ$

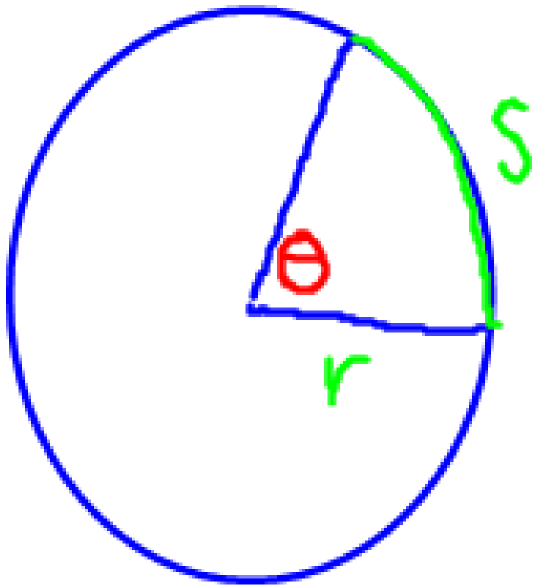
$= 37.41^\circ$

7 D → DMS

$$58.26^\circ = \underline{58}^\circ\ \underline{15}'\ \underline{36}''$$

$$0.26^\circ \cdot \frac{60'}{1^\circ} = 15.\underline{\underline{6}}'$$

$$.6' \cdot \frac{60''}{1'} = 36''$$

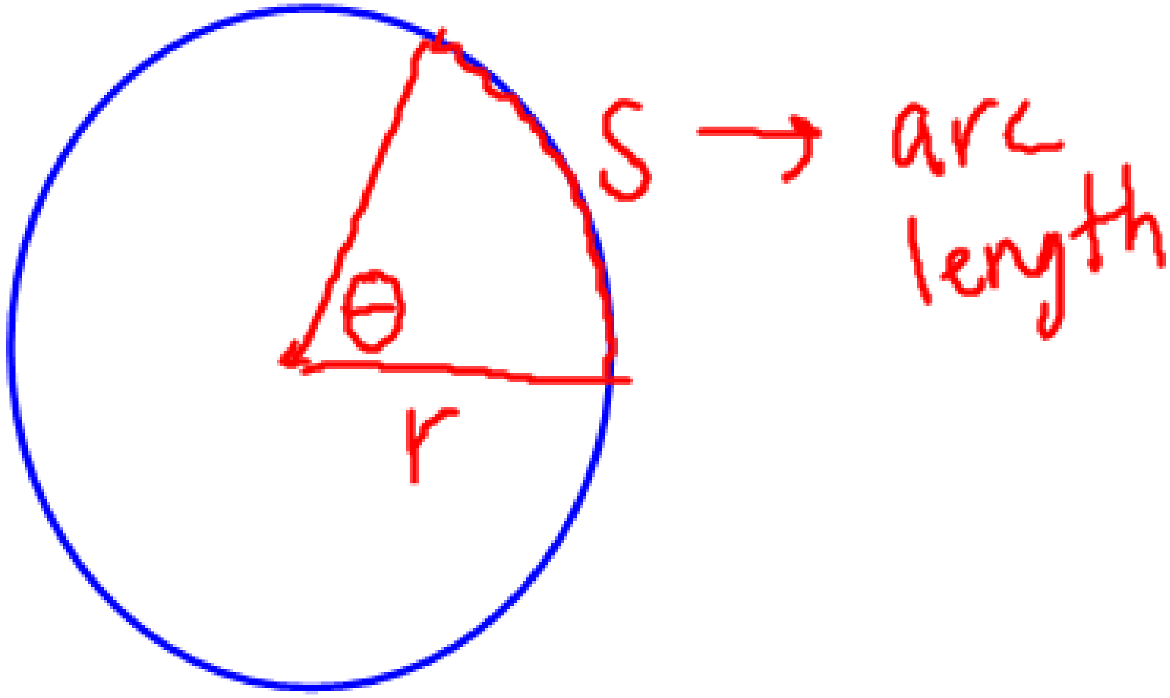


$\theta \rightarrow$ central angle

$s \rightarrow$ arc length

$$\frac{\theta}{360} = \frac{s}{C}$$

$$\frac{\theta}{360} = \frac{s}{2\pi r}$$



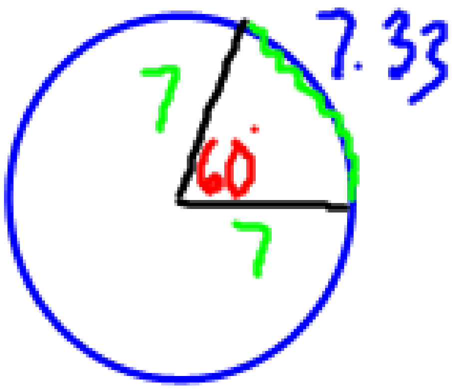
$$\frac{\theta}{360} = \frac{S}{C}$$

$$\frac{\theta}{360} = \frac{s}{2\pi r}$$

8

EXAMPLE 3 Perimeter of a Pizza Slice

Find the perimeter of a 60° slice of a large (7-in. radius) pizza.



$$\frac{60^\circ}{360} = \frac{s}{14\pi}$$

$$\frac{360s}{360} = \frac{840\pi}{360}$$

$$s = \frac{7}{3}\pi$$

$$s \approx 7.33$$

$$P = 21.33 \text{ in.}$$

9. Find the radius of a circle if the central angle of 100 degrees cuts an arc length of 20 inches.

$$\frac{100}{360} = \frac{20}{2\pi r}$$

$$r \approx 11.46 \text{ in.}$$

$$\frac{200\pi r}{200\pi} = \frac{7200}{200\pi}$$

$$r = \frac{36}{\pi}$$

10. Find the arc length of an arc formed by a central angle of 4 radians with a radius of 6 centimeters.

$$\frac{\theta}{2\pi} = \frac{s}{2\pi r}$$

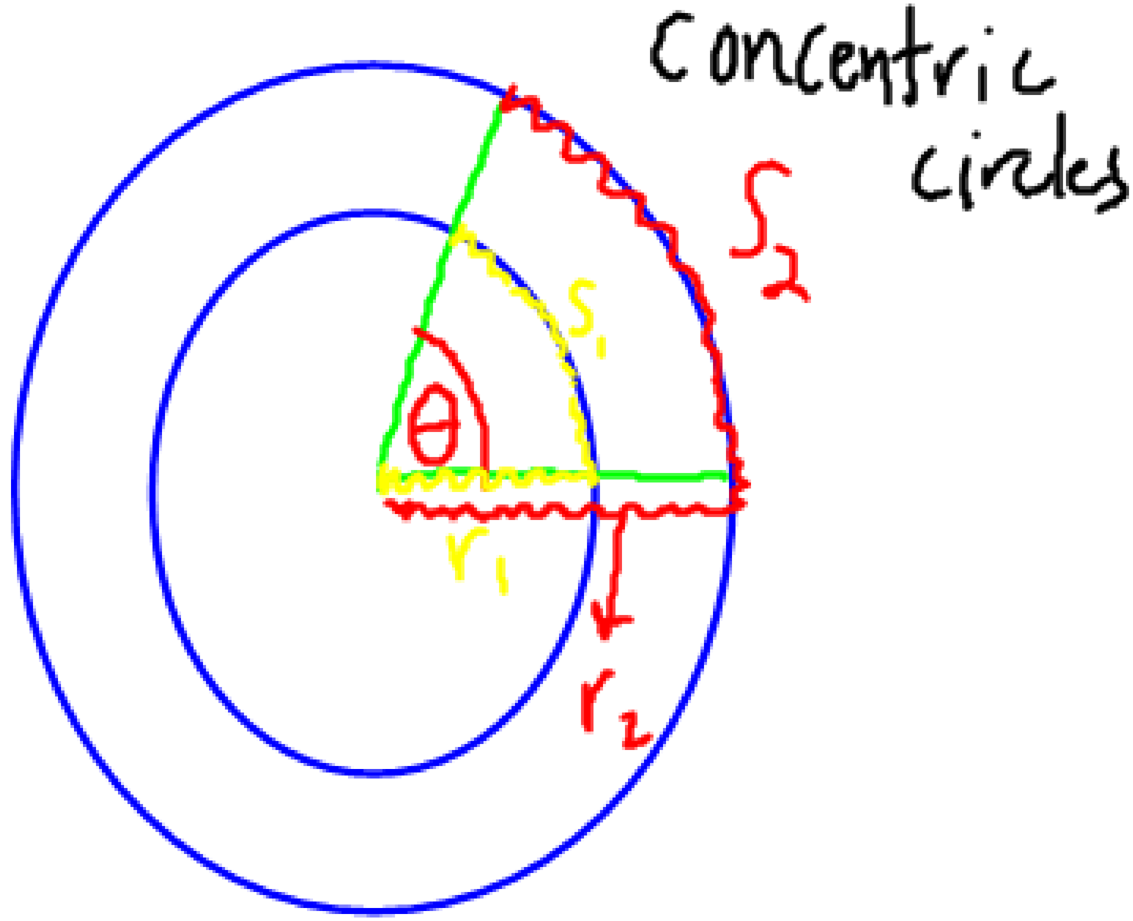
$$\theta = \frac{s}{r}$$

only if θ = radians

$$4 = \frac{s}{6}$$

$$s = 24 \text{ cm}$$

$$\theta = \frac{s_1}{r_1}$$



$$\theta = \frac{s_2}{r_2}$$

$$\frac{s_1}{r_1} = \frac{s_2}{r_2}$$

11. For two concentric circles, if the r = 6 and s = 8 for the smaller circle, find s for the larger if r = 15 for the larger circle.

$$\frac{8}{6} = \frac{s}{15}$$

$$s = 20$$

Angular and Linear Motion

In applications it is sometimes necessary to connect *angular speed* (measured in units like revolutions per minute) to *linear speed* (measured in units like miles per hour). The connection is usually provided by one of the arc length formulas or by a conversion factor that equates "1 radian" of angular measure to "1 radius" of arc length.

EXAMPLE 5 Using Angular Speed

Albert Juarez's truck has wheels 36 inches in diameter. If the wheels are rotating at 630 rpm (revolutions per minute), find the truck's speed in miles per hour.

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$$\frac{630\,\cancel{\text{rev}}}{\cancel{\text{min}}} \cdot \frac{60\,\cancel{\text{min}}}{1\,\text{hr}} \cdot \frac{36\pi\,\cancel{\text{in}}}{1\,\cancel{\text{rev}}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{mile}}{5280\,\cancel{\text{ft}}} \quad \frac{\text{mi}}{\text{hr}}$$

$$\frac{(630)(60)(36\pi)}{12(5280)} \; \frac{\text{mi}}{\text{hr}} = 67.47\ \text{mph}$$

$1\ \text{rev} = C = 36\pi$

A **nautical mile** mile (naut mi) is the length of 1 minute of arc along Earth's equator. Figure 4.6 shows, though not to scale, a central angle *AOB* of Earth that measures 1/60 of a degree. It intercepts an arc 1 naut mi long.

The arc length formula allows us to convert between nautical miles and **statute miles** (stat mi), the familiar "land mile" of 5280 feet.

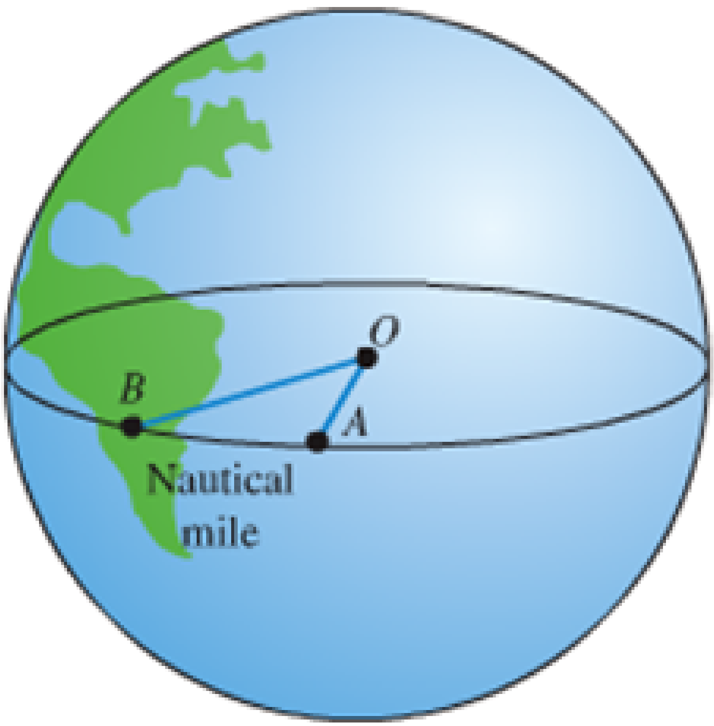

FIGURE 4.6 Although Earth is not a perfect sphere, its diameter is, on average, 7912.18 statute miles. A nautical mile is 1′ of Earth's circumference at the equator.

Convert 1 minute to radians:

$$1' = \left(\frac{1}{60}\right)^{\circ} \times \frac{\pi \text{ rad}}{180^{\circ}} = \frac{\pi}{10{,}800} \text{ radians}$$

Now we can apply the formula $s = r\theta$:

$$\begin{aligned} 1 \text{ naut mi} &= (3956)\left(\frac{\pi}{10{,}800}\right) \text{ stat mi} \\ &\approx 1.15 \text{ stat mi} \\ 1 \text{ stat mi} &= \left(\frac{10{,}800}{3956\pi}\right) \text{ naut mi} \\ &\approx 0.87 \text{ naut mi} \end{aligned}$$

Distance Conversions

$$\begin{aligned} 1 \text{ statute mile} &\approx 0.87 \text{ nautical mile} \\ 1 \text{ nautical mile} &\approx 1.15 \text{ statute miles} \end{aligned}$$

Megan McCarty, a pilot for Western Airlines, frequently pilots flights from Boston to San Francisco, a distance of 2698 stat mi. Captain McCarty's calculations of flight time are based on nautical miles. How many nautical miles is it from Boston to San Francisco?

$$2698 \text{ stat mi} \cdot \frac{0.87 \text{ naut mile}}{1 \text{ stat mile}} \approx 2347.26$$

nautical miles