3.7 Implicit Differentiation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

r^2 r^2

$$x^2 + y^2 = r^2$$

circle center (0,0)

$r \rightarrow$ radius

① $x^2 + y^2 = 9$. Find $\frac{dy}{dx}$

Take derivative of both sides with respect to x.

$$x^2 + y^2 = 9$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{2y\,\frac{dy}{dx}}{2y} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Find eqn of tangent line at $(2, \sqrt{5})$

$$\frac{dy}{dx} = m = \frac{-2}{\sqrt{5}}$$

$$y = \frac{-2}{\sqrt{5}}x + \sqrt{5} + \frac{4}{\sqrt{5}}$$

$$\sqrt{5} = -\frac{2}{\sqrt{5}} \cdot 2 + b$$

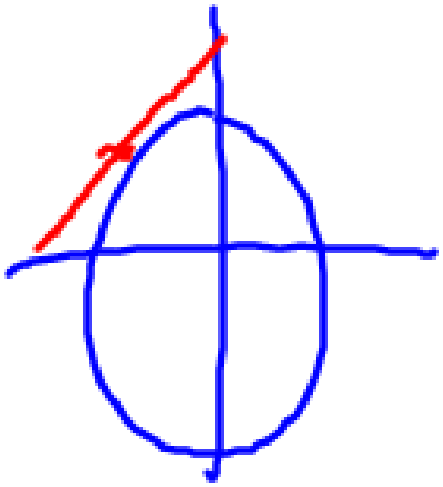
$$\sqrt{5} + \frac{4}{\sqrt{5}} = b$$

② $x^2 + xy - 3y^2 = 1$ Find $\frac{dy}{dx}$

$2x + 1 \cdot y + x\frac{dy}{dx} - 6y \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx}(x - 6y) = -2x - y$

$\frac{dy}{dx} = \frac{-2x - y}{x - 6y}$



(9) $y^2 = \frac{x-1}{x+1}$ Find $\frac{dy}{dx}$

$$2y\frac{dy}{dx} = \frac{1(x+1) - 1(x-1)}{(x+1)^2}$$

$$\frac{1}{2y} \cdot 2y\frac{dy}{dx} = \frac{2}{(x+1)^2} \cdot \frac{1}{2y}$$

$$\frac{dy}{dx} = \frac{1}{y(x+1)^2}$$

Find normal line equation at $(3, 2)$

$$m = \frac{1}{2(3+1)^2} = \frac{1}{32}$$

$m = -32$ $\quad (3, 2)$

$2 = (-32)(3) + b$

$98 = b$

$y = -32x + 98$

④ $x = \tan y$ Find $\frac{dy}{dx}$

$1 = \sec^2 y \cdot \frac{dy}{dx}$

$\frac{1}{\sec^2 y} = \frac{dy}{dx}$

$\cos^2 y = \frac{dy}{dx}$

(5) $x + \sin 4y = xy$ Find $\frac{dy}{dx}$

$$1 + (\cos 4y) \cdot 4 \cdot \frac{dy}{dx} = 1 \cdot y + x \cdot \frac{dy}{dx}$$

$$1 + 4\cos 4y \cdot \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$4\cos 4y \cdot \frac{dy}{dx} - x \frac{dy}{dx} = y - 1$$

$$\frac{dy}{dx}\left(4\cos 4y - x\right) = y - 1$$

$$\frac{dy}{dx} = \frac{y-1}{4\cos 4y - x}$$

⑥ $12x^3 - 3y^2 = 8$ Find $\frac{d^2y}{dx^2}$ → 2nd derivative.

$$6x^2 - 6y \cdot \frac{dy}{dx} = 0$$

$$6x^2 = 6y \frac{dy}{dx}$$

$$\frac{x^2}{y} = \frac{dy}{dx}$$

$$\frac{x^2}{y} = \frac{dy}{dx}$$

$$\frac{2xy - x^2\frac{dy}{dx}}{y^2} = \frac{d^2y}{dx}$$

$$\frac{2xy - \frac{x^2}{y}}{y^2} = \frac{d^2y}{dx^2}$$

$$\frac{2xy - \frac{x^4}{y}}{y^2} = \frac{d^2y}{dx^2}$$

$$\frac{\frac{2xy}{y^2} - \frac{x^4}{y^2}}{y}$$

$$\frac{2x}{y} - \frac{x^4}{y^3} = \frac{d^2y}{dx^2}$$