

Section 3.4 Notes: Linear Programming

Name: _____

feasible region \rightarrow possible solutions

constraint \rightarrow restriction / limitation

objective function \rightarrow function you are trying to maximize or minimize

- What point in the feasible region maximizes P for the objective function $P = 10x + 15y$? What point minimizes P ?

$$3x = 60 \\ x = 20$$

$$6y = 60 \\ y = 10$$

Constraints

$$\begin{cases} x + y \leq 16 \\ 3x + 6y \leq 60 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

Graph using intercepts

Step 1

Graph the constraints and shade the feasible region.

Step 2

Find the coordinates for each vertex of the region.

$$P = 10x + 15y$$

max/min points will occur at a vertex of feasible region

Step 3

Evaluate P at each vertex.

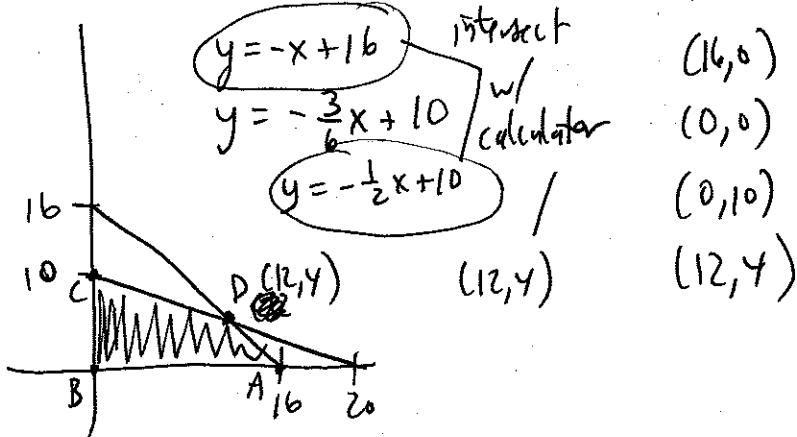
~~$P = 10x + 15y$~~

$$P = 10(16) + 15(0) = 160$$

$$P = 10(0) + 15(0) = 0$$

$$P = 10(0) + 15(10) = 150$$

$$P = 10(12) + 15(4) = 180$$



The maximum value of the objective function is 180. It occurs when $x = 12$ and $y = 4$.

The minimum value of the objective function is 0. It occurs when $x = 0$ and $y = 0$.

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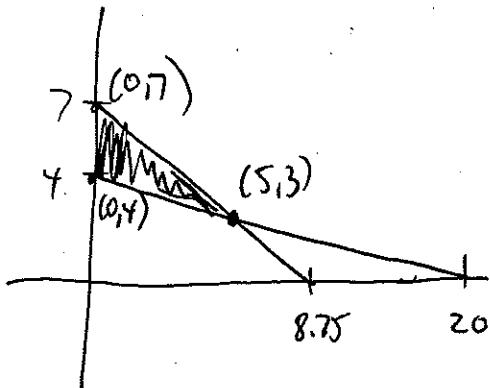
2. Graph each system of constraints. Name all vertices. Then find the values of x and y that maximize or minimize the objective function.

$$\begin{aligned}
 y &= \frac{35}{7} = 5 \\
 x &= \frac{35}{4} = 8.75 \\
 y &= \frac{20}{5} = 4 \\
 y &= 20 \\
 y &= \frac{35 - 5x}{5} = 7 - x \\
 y &= \frac{20 - x}{5} = 4 - \frac{x}{5}
 \end{aligned}
 \quad \left\{
 \begin{array}{l}
 5y + 4x \leq 35 \quad B \\
 5y + x \geq 20 \quad A \\
 y \geq 0 \quad C \\
 x \geq 0 \quad D
 \end{array}
 \right. \quad \left. \begin{array}{l}
 y = -\frac{4}{5}x + 7 \\
 y = -\frac{x}{5} + 4
 \end{array} \right\} \text{Intercept w/ calc} \rightarrow (5, 3)$$

$$y = 20$$

Step 1

Graph the constraints and shade the feasible region.



Step 2

Find the coordinates for each vertex of the region.

VERTEX

(0, 4)
(0, 7)
(5, 3)

Step 3

Evaluate P at each vertex.

$$P = 8x + 2y$$

$$P = 8(0) + 2(4) = 8$$

$$P = 8(0) + 2(7) = 14$$

$$P = 8(5) + 2(3) = 46$$

The maximum value of the objective function is 46. It occurs when $x = 5$ and $y = 3$.

The minimum value of the objective function is 8. It occurs when $x = 0$ and $y = 4$.

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3. Your school band is selling calendars as a fundraiser. Wall calendars cost \$48 per case of 24. You sell them at \$7 per calendar. Pocket calendars cost \$30 per case of 40. You sell them at \$3 per calendar. You make a profit of \$120 per case of wall calendars and \$90 per case of pocket calendars. If the band can buy no more than 1000 total calendars and spend no more than \$1200, how can you maximize your profit if you sell every calendar? What is the maximum profit?

Define Let x = number of cases of wall calendars
Let y = number of cases of pocket calendars

Relate Organize the information in a table.

	Wall Calendars	Pocket Calendars	Total
Number of Cases			
Number of Units			
Cost			
Profit			

Write Use the information in the table and the definitions of x and y to write the constraints and the objective function. Simplify the inequalities if necessary.

Constraints

$$\begin{cases} x \geq 0 \\ y \geq 0 \end{cases} \quad \begin{array}{l} \text{can't sell} \\ \text{negative #} \\ \text{of calendars} \end{array}$$

$$\text{Total cost} \leq 1200$$

$$48x + 30y \leq 1200$$

$$y = -\frac{48}{30}x + 40$$

$$\text{Total # of calendar} \leq 1000$$

$$24x + 40y \leq 1000$$

$$y = -\frac{24}{40}x + 25$$

Step 1

Graph the constraints and shade to see the feasible region

$$\text{Objective function: } P = 120x + 90y$$

Step 2

Find the coordinates for each vertex of the region.

Step 3

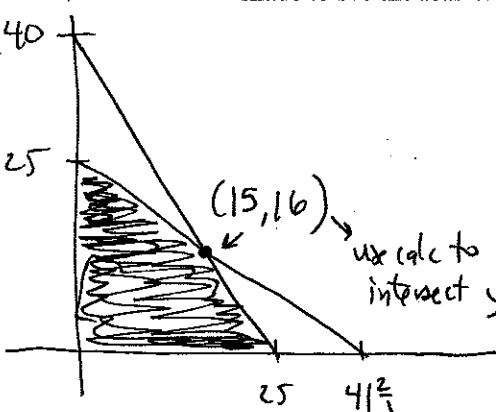
Evaluate the objective function using the vertex coordinates.

$$(0, 25) \quad P = 120(0) + 90(25) = 2250$$

$$(0, 0) \quad P = 120(0) + 90(0) = 0$$

$$(25, 0) \quad P = 120(25) + 90(0) = 3000$$

$$(15, 16) \quad P = 120(15) + 90(16) = 3240$$



use calc to
intersect $y =$ lines
above

You can maximize your profit by selling 15 cases of wall calendars and 16 cases of pocket calendars. The maximum profit is \$3240.

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4. A glass blower can form 8 simple vases or 2 elaborate vases in an hour. In a work shift of no more than 8 hours, the worker must form at least 40 total vases. If the glass blower makes a profit of \$30 per hour on simple vases and \$35 per hour on the elaborate vases, how many hours should this person spend on each type of vase in order to maximize profit?

$x = \# \text{ hrs spent on simple vase}$

$y = \# \text{ hrs spent on elaborate vase}$

Total Hours ≤ 8

$$\boxed{x+y \leq 8} \rightarrow y = -x+8$$

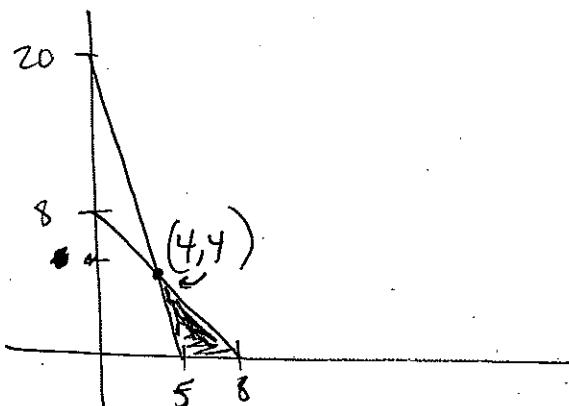
Total Vases ≥ 40

$$\boxed{8x+2y \geq 40} \quad y = -4x + 20$$

$$\begin{cases} x \geq 0 \\ y \geq 0 \end{cases}$$

$$P = 30x + 35y$$

Vertices



$$\begin{aligned} (5,0) \quad P &= 30(5) + 35(0) = 150 \\ (8,0) \quad P &= 30(8) + 35(0) = 240 \\ (4,4) \quad P &= 30(4) + 35(4) = 260. \end{aligned}$$

Max Profit of \$260

when 4 hrs each are spent
making simple and elaborate vases.