u-Substitution - Homework

1. $\int \sqrt{x-2}\,dx$

$u = x-2$

$\frac{du}{dx} = 1$

$du = dx$

$u^{1/2}\,du$

$\frac{2}{3}u^{3/2} + c$

$\frac{2}{3}(x-3)^{3/2} + C$

$\frac{2}{3}\sqrt[2]{(x-3)^3} + c$

2. $\int (2x+3)^{11}\,dx$

$u = 2x+3$

$\frac{du}{dx} = 2$

$\frac{du}{2} = dx$

$\frac{1}{2}\int u^{11} \cdot du$

$\frac{1}{2} \cdot \frac{u^{12}}{12}$

$\frac{u^{12}}{24} + c \rightarrow \frac{(2x+3)^{12}}{24} + C$

3. $\int \sqrt{5x-1}\,dx$

$u = 5x-1$

$\frac{du}{dx} = 5$

$\frac{du}{5} = dx$

$\frac{1}{5}\int \sqrt{u}\,du$

$\frac{1}{5} \cdot \frac{2u^{3/2}}{3} + C$

$\frac{2}{15}(5x-1)^{3/2} + C$

4. $\int \sqrt[3]{6x+1}\,dx$

$u = 6x+1$

$\frac{du}{dx} = 6 \rightarrow \frac{du}{6} = dx$

$\frac{1}{6}\int u^{1/3}\,du$

$\frac{1}{6} \cdot \frac{3}{4}u^{4/3}$

$\frac{1}{8}(6x+1)^{4/3} + C$

5. $\int 5(3-4x)^{2/3}\,dx$

$u = 3-4x$

$\frac{du}{dx} = -4$

$\frac{du}{-4} = dx$

$\frac{5}{-4}\int u^{2/3}\,du$

$-\frac{3}{4}(3-4x)^{5/3} + C$

6. $\int \frac{dx}{(8x-1)^3}$

$u = 8x-1$

$\frac{1}{8}\int \frac{du}{u^3}$

$-\frac{1}{16}u^{-2}$

$-\frac{1}{16(8x-1)^2} + C$

7. $\int x(x^2+2)^6\,dx$

$u = x^2+2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int u^6\,du$

$\frac{(x^2+2)^7}{14} + C$

8. $\int 6x^2\sqrt{3x^3-1}\,dx$

$u = 3x^3+1$

$\frac{du}{dx} = 9x^2$

$\frac{du}{9} = x^2\,dx$

$\frac{6}{9}\int u^{1/2}\,du$

$\frac{2}{3} \cdot \frac{2}{3}u^{3/2} + c$

$\frac{4}{9}(3x^3-1)^{3/2} + C$

9. $\int \left(1+\frac{1}{x}\right)^3\left(\frac{1}{x^2}\right)dx$

$u = 1+\frac{1}{x}$

$\frac{du}{dx} = -\frac{1}{x^2}$

$-du = dx \cdot \frac{1}{x^2}$

$-\int u^3\,du$

$-\frac{(1+\frac{1}{x})^4}{4} + C$

10. $\int x^{1/3}\left(x^{4/3}+9\right)^8 dx$

$u = x^{4/3}+9$

$\frac{du}{dx} = \frac{4}{3}x^{1/3}$

$\frac{3}{4}du = x^{1/3}\,dx$

$\frac{3}{4}\int u^8\,du$

$\frac{(x^{4/3}+9)^9}{12} + C$

11. $\frac{2}{3}\int \sqrt{4-\frac{3}{5}x}\,dx$

$u = 4-\frac{3}{5}x$

$\frac{du}{dx} = -\frac{3}{5}$

$-\frac{5}{3}du = dx$

$-\frac{5}{3} \cdot \frac{2}{3}\int \sqrt{u}\,du$

$-\frac{5}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}u^{3/2}$

$-\frac{20}{27}\left(4-\frac{3}{5}x\right)^{3/2} + C$

12. $\int (3x+15)\sqrt{x^2+10x+4}\,dx$

$u = x^2+10x+4$

$\frac{du}{dx} = 2x+10$

$2(x+5)$

$\frac{du}{2} = (x+5)dx$

$3\int (x+5)$

$3 \cdot \frac{1}{2}\int u^{1/2}\,du$

$\frac{3}{2} \cdot \frac{2}{3}u^{3/2} + C$

$u^{3/2} + C$

$(x^2+10x+4)^{3/2} + C$

13. $\int (x+2)\sqrt{x-2}\,dx$

14. $\int \frac{x^2}{\sqrt{x-4}}\,dx$

15. $\int \sin 5x\,dx$

$u = 5x$

$\frac{du}{dx} = 5$

$\frac{du}{5} = dx$

$\frac{1}{5}\int \sin u\,du$

$-\frac{1}{5}\cos u + C$

$-\frac{1}{5}\cos 5x + C$

16. $\int \cos\frac{x}{2}\,dx$

$u = \frac{x}{2}$

$\frac{du}{dx} = \frac{1}{2}$

$dx = 2\,du$

$2\int \cos u\,du$

$2\sin u + C$

$2\sin\frac{x}{2} + C$

17. $\int \frac{1}{3}\sec^2 8x\,dx$

$u = 8x$

$\frac{du}{dx} = 8$

$\frac{du}{8} = dx$

$\frac{1}{3}\cdot\frac{1}{8}\int \sec^2 u\,du$

$\frac{1}{24}\tan(8x) + C$

18. $\int \sin 4x \cos 4x\,dx$

$u = \sin 4x$

$\frac{du}{dx} = \cos 4x \cdot 4$

$\frac{du}{4} = \cos 4x\,dx$

$\frac{1}{4}\int u\,du$

$\frac{1}{4}\sin 4x + C$

19. $\int \cos^3 x \sin x\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-du = \sin x\,dx$

$-\int u^3\,du$

$-\frac{\cos^4 x}{4} + C$

20. $\int \tan x \sec^2 x\,dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\,dx$

$\int u\,du$

$\frac{u^2}{2} + C$

$\frac{\tan^2 x}{2} + C$

21. $\int \sqrt{\cos 6x}\sin 6x\,dx$

$u = \cos 6x$

$\frac{du}{dx} = 6\sin 6x$

$\frac{du}{6} = \sin 6x\,dx$

$\frac{1}{6}\int u^{1/2}\,du$

$\frac{1}{6}\cdot\frac{2}{3}\cdot u^{3/2} + C$

$\boxed{\frac{1}{9}(\cos 6x)^{3/2} + C}$

22. $\int \frac{\sin x}{(4-\cos x)^3}\,dx$

$u = 4 - \cos x$

$\frac{du}{dx} = \sin x$

$du = \sin x\,dx$

$\int \frac{du}{u^3}$

$\int u^{-3}\,du$

$\frac{u^{-2}}{-2} + C$

$-\frac{1}{2u^2} + C$

$-\frac{1}{2(4-\cos x)^2}$

Definite Integration with u-Substitution - Homework

Find the values of the following definite integrals. Verify using your calculator. Some will use *u*-substitution, others will not.

1. $\int_{-2}^{2}\left(x^3-1\right)dx$

$\frac{x^4}{4}-x\Big]_{-2}^{2}$

-4

2. $\int_{0}^{4}x\left(\sqrt{x}-1\right)dx$

$x(x^{1/2}-1)$

$\int_0^4\left(x^{3/2}-x\right)dx$

$\frac{2}{5}x^{5/2}-\frac{x^2}{2}\Big]_0^4$

$\frac{24}{5}=4.8$

3. $\int_{0}^{\pi/3}\sin(2x)\,dx$

$u=2x$

$\frac{du}{dx}=2$

$\frac{du}{2}=dx$

$\frac{1}{2}\int\sin u\,du$

$-\frac{1}{2}\cos u\Big]_0^{\frac{2}{3}\pi}$

$-\frac{1}{2}\left[\cos\frac{2}{3}\pi-\cos 0\right]$ $-\frac{1}{2}\cdot-\frac{3}{2}$

$-\frac{1}{2}\left[-\frac{1}{2}-1\right]$

$+\frac{3}{4}$

4. $\int_{0}^{\pi/12}\left(1-\cos 2x\right)dx$

$\int_0^{\pi/12}1\,dx-\int_0^{\pi/12}\cos 2x$ $\int u^2\,du$

$x\Big]_0^{\pi/12}-\frac{1}{2}\sin 2x\Big]_0^{\pi/12}$ $\frac{u^3}{3}\Big]_1^2$

$\frac{\pi}{12}-\frac{1}{4}=.011799$ $\frac{8}{3}-\frac{1}{3}=\frac{7}{3}$

5. $\int_{0}^{1}2x\left(x^2+1\right)^2dx$

$u=x^2+1$

$\frac{du}{dx}=2x$

$du=2x\,dx$

6. $\int_{0}^{3}x\sqrt{9-x^2}\,dx$

$u=9-x^2$

$\frac{du}{dx}=-2x$

$\frac{du}{-2}=x\,dx$

$-\frac{1}{2}\int u^{1/2}\,du$

$-\frac{1}{2}\cdot\frac{2}{3}\cdot u^{3/2}$

$-\frac{1}{3}u^{3/2}\Big]_9^0$

$\frac{1}{3}u^{3/2}\Big]_0^9$ $\frac{1}{3}[27-0]=9$

7. $\int_{0}^{5}\left|x-4\right|dx$

8. $\int_{0}^{4}\left|x-\sqrt{x}\right|dx$

9. $\int_{2}^{3}\frac{x}{\left(x^2-3\right)^2}dx$

$u=x^2-3$

$\frac{du}{dx}=2x$

$\frac{du}{2}=x\,dx$

$\frac{1}{2}\int\frac{1}{u^2}\,du$

$u^{-2}\,du$

$-\frac{1}{2}u^{-1}\Big]_1^6$ $-\frac{1}{2}\left[\frac{1}{6}-1\right]$

$-\frac{1}{2}\cdot-\frac{5}{6}=\frac{5}{12}$

10. $\int_{0}^{4}\frac{dt}{\sqrt{2t+1}}$

$u=2t+1$

$\frac{du}{dt}=2$

$\frac{du}{2}=dt$

$\frac{1}{2}\int_0^4\frac{du}{\sqrt{u}}$

$u^{1/2}\Big]_0^4$

$\sqrt{2t+1}\Big]_0^4$

$3-1$

2

11. $\int_{0}^{\pi/2}\cos^3 t\sin t\,dt$

$u=\cos t$

$\frac{du}{dt}=-\sin t$

$-du=\sin t\,dt$

$-\int u^3\,du$

$-\frac{u^4}{4}\Big]_1^0$

$\frac{u^4}{4}\Big]_0^1$

$\frac{1}{4}$

12. $\int_{0}^{\sqrt{\pi/2}}t\sin\left(\pi-t^2\right)dt$

$u=\pi-t^2$

$\frac{du}{dt}=-2t$

$\frac{du}{-2}\,t\,dt$

$-\frac{1}{2}\int\sin u\,du$

$+\frac{1}{2}\cos u$

$\frac{1}{2}\cos u\Big]_0^{\sqrt{\pi/2}}$

$\frac{1}{2}\cos(\pi-t^2)\Big]_0^{\sqrt{\pi/2}}$

$0+\frac{1}{2}$

$\frac{1}{2}$

13. $\int_0^{\pi/4} \sqrt{\tan x}\,\sec^2 x\,dx$

$u = \tan x$

$\frac{2}{3}$

14. $\int_0^{\pi/3} \cos x\sqrt{1-\cos^2 x}\,dx$

$u = 1-\cos^2 x$

$\frac{du}{dx} = 2\cos x$

$\frac{du}{2} = \cos x\,dx$

$\frac{1}{2}\int \sqrt{1-u}\,du$

$\frac{1}{2}\int (1-u)^{1/2}\,du$

$-\frac{1}{2}\cdot\frac{2}{3}(1-u)^{3/2} + C\Big]_0^{\pi/3}$

$-\frac{1}{3}(1-\cos^2 x)^{3/2}\Big]_0^{\pi/3}$

15. $\int_0^1 x\sqrt{ax^2+b}\,dx$

16. $\int_{\pi^2/4}^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$ $\quad x^{1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$2\int \frac{\sin u}{u}\,du$

17. $\int_0^4 |9-x^2|\,dx$

18. $\int_{-4}^{4} \frac{1}{x^2}\,dx$ (Be careful!)

$\int x^{-2}$

$-x^{-1}$

$-\frac{1}{x}\Big]_{-4}^{4}$

$-\frac{1}{4} - -\frac{1}{-4}$

0

If $\int_0^2 f(x)\,dx = \frac{11}{3}$ and $\int_0^6 f(x)\,dx = 15$, $f(x)$ is an even function (symmetric to the y - axis), find the following:

19. $\int_{-2}^{0} f(x)\,dx$ 20. $\int_{-2}^{2} f(x)\,dx$ 21. $\int_0^2 -f(x)\,dx$ 22. $\int_{-2}^{0} 3f(x)\,dx$ 23. $\int_0^2 f(3x)\,dx$

If $\int_0^2 f(x)\,dx = \frac{11}{3}$ and $\int_0^6 f(x)\,dx = 15$, $f(x)$ is an odd function (symmetric to the origin), find the following:

24. $\int_{-2}^{0} f(x)\,dx$ 25. $\int_{-2}^{2} f(x)\,dx$ 26. $\int_0^2 -f(x)\,dx$ 27. $\int_{-2}^{0} 3f(x)\,dx$ 28. $\int_{-2}^{2} f(3x)\,dx$