Profit = Revenue − Expenses

$P = 20x - (8x + 10{,}000)$

$P = 12x - 10{,}000$

$20

x = # of units

Misc Exp = 10,000

6.6 FUNCTION OPERATIONS

take note

Key Concepts Function Operations

Addition	$(f + g)(x) = f(x) + g(x)$
Subtraction	$(f - g)(x) = f(x) - g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of *both* f and g. Also, the domain of the quotient function does not contain any x-value for which $g(x) = 0$.

Ex. 1 Adding and Subtracting functions: Let Let $f(x) = 2x^2 + 8x + 5$ and $g(x) = x - 3$.

What are $f + g$ and $f - g$ and their domains?

$(f+g)(x) = (2x^2+8x+5)+(x-3)$

$(f+g)(x) = 2x^2+9x+2$ D: all real #'s

$(-\infty, \infty)$

$(f-g)(x) = (2x^2+8x+5)-(x-3)$

$-x+3$

$(f+g)(x) = 2x^2+7x+8$ D: $(-\infty, \infty)$ $\mathbb{R}$

Ex. 2 Adding and Subtracting functions: $f(x) = 4x + 7$ and $g(x) = \sqrt{x} + 3$. What are $f + g$ and $f - g$ and their domains?

$(f+g)(x) = (4x+7) + (\sqrt{x}+3)$

$(f+g)(x) = 4x + \sqrt{x} + 10$ D: $x \geq 0$ $[0, \infty)$

$(f-g)(x) = (4x+7) - (\sqrt{x}+3)$

$= 4x - \sqrt{x} + 4$

Ex. 3 Multiplying and Dividing functions: Let $f(x) = x - 9$ and $g(x) = x + 4$. What are $f \cdot g$ and $\frac{f}{g}$ and $\frac{g}{f}$ their domains?

D: $(-\infty, \infty)$

$(f \cdot g)x = (x-9)(x+4) = x^2 - 5x - 36$

$\left(\frac{f}{g}\right)(x) = \frac{x-9}{x+4}$

$x + 4 \neq 0$

D: $x \neq -4$

$(-\infty, -4) \cup (-4, \infty)$

$$\left(\frac{g}{f}\right)(x) = \frac{x+4}{x-9}$$

$x - 9 \neq 0$

$D: x \neq 9$

$(-\infty, 9) \cup (9, \infty)$

Ex. 4 Multiplying and Dividing functions: Let $f(x) = 3x^2 - 11x - 4$ and $g(x) = 3x + 1$.

$f \cdot g$ and $\frac{f}{g}$ and $\frac{g}{f}$ their domains?

$(f \cdot g)(x) = (3x+1)(3x^2-11x-4)$

$9x^3 - 33x^2 - 12x$

$+ 3x^2 - 11x - 4$

$9x^3 - 30x^2 - 23x - 4$

D: all real #'s

$$\left(\frac{f}{g}\right)(x) = \frac{3x^2-11x-4}{3x+1}$$

$3x+1 \neq 0$

all reals except

D: $x \neq -\frac{1}{3}$

$$\left(\frac{g}{f}\right)(x) = \frac{3x+1}{3x^2-11x-4}$$

$3x^2-11x-4 \neq 0$

$(3x+1)(x-4) \neq 0$

$3x+1 \neq 0 \quad x-4 \neq 0$

D: All reals but $x \neq -\frac{1}{3}, x \neq 4$

COMPOSITE FUNCTION:

take note

Key Concept Composition of Functions

The composition of function g with function f is written as $g \circ f$ and is defined as $(g \circ f)(x) = g(f(x))$. The domain of $g \circ f$ consists of the x-values in the domain of f for which $f(x)$ is in the domain of g.

$(g \circ f)(x) = g(f(x))$

1. Evaluate $f(x)$ first.
2. Then use $f(x)$ as the input for g.

Function composition is not commutative since $f(g(x))$ does not always equal $g(f(x))$.

Ex. 5 Composing Functions: Let $f(x) = x - 5$ and $g(x) = x^2$. What is $(g \circ f)(-3)$?

$g(f(-3)) = g(-8) = (-8)^2 = 64$

$f(-3) = -3 - 5 = -8$

Ex. 6 Composing Functions: Let $f(x) = x - 5$ and $g(x) = x^2$. What is $(f \circ g)(-3)$?

$f(g(-3)) = f(9) = 9 - 5 = 4$

$g(-3) = (-3)^2 = 9$

Ex. 7 Composing functions: Let $f(x) = \sqrt{x-5}$ and $g(x) = x + 3$. What is $f(g(6))$?

$f(g(6)) = f(9) = \sqrt{9-5} = \sqrt{4} = 2$

$g(6) = 6 + 3 = 9$

Ex. 8 Composing functions: Let $f(x) = \sqrt{x-5}$ and $g(x) = x + 3$. What is $g(f(21))$?

$g(f(21)) = g(4) = 4 + 3 = 7$

$f(21) = \sqrt{21-5} = \sqrt{16} = 4$

Ex. 9 Composing functions: Let $f(x) = 3x + 4$ and $g(x) = x - 5$. What is $f(g(x))$?

$f(g(x)) = f(x-5) = 3(x-5)+4$

$= 3x - 15 + 4$

$f(g(x)) = 3x - 11$

Ex. 10 Composing functions: Let $f(x) = 3x + 4$ and $g(x) = x - 5$. What is $(g \circ f)(x)$?

$g(f(x)) = g(3x+4) = (3x+4) - 5$

$g(f(x)) = 3x - 1$

Ex. 11 You have a coupon good for $5 off the price of any large pizza. You also get a 10% discount on any pizza if you show your student ID. How much more would you pay for a large pizza if the cashier applies the coupon first?

Know

The coupon value and the discount rate

Need

The difference between the results of applying the discount or coupon first

Plan

- Compose two functions in two ways.
- Then find the difference in their results.

original

x = price of pizza

$5 off

$f(x) = x - 5$

10% off

$g(x) = .90x$

$f(x) = x - 5$

$g(x) = 0.90x$

10%, then $5

$f(g(x))$

$f(.90x)$

$.90x - 5$

$.50 saved if

$5 off, then 10%

$g(f(x))$

$g(x-5)$

$.90(x-5)$

$.90x - 4.50$