

6.5 Solving Radical Equations

$$\textcircled{1} \sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

$$\textcircled{2} (\sqrt{x})^2 = (3)^2$$

$$x = 9$$

$$\textcircled{3} \left(\sqrt{3x-2} \right)^2 = (7)^2$$

$$3x-2 = 49$$

+2 +2

$$\frac{3x}{3} = \frac{51}{3}$$

$$x = 17$$

$$\textcircled{4} \quad 3\sqrt{5-x} + 2 = 8$$

$$3y + \frac{2}{1} = 8$$

$$3y = \frac{6}{3}$$

$$y = 2$$

$$\left(\sqrt{5-x}\right)^2 = (2)^2$$

$$5 - x = 4$$

$$-x = -1$$

$$\textcircled{x=1}$$

$$\textcircled{5} \quad 5 \sqrt{2x-7} - 3 = 42$$

$+3 \qquad \qquad +3$

$$\frac{5\sqrt{2x-7}}{5} = \frac{45}{5}$$

$$\left(\sqrt{2x-7}\right)^2 = \left(9\right)^2$$

$$2x - 7 = 81$$

$$+7 \quad +7$$

$$\frac{2x}{2} = \frac{88}{2}$$

$$x = 44$$

$$\textcircled{6} \quad (\sqrt{5x-7})^2 = (\sqrt{11x-28})^2$$

$$\begin{array}{r} 5x-7 = 11x-28 \\ -5x \quad \quad -5x \end{array}$$

$$\begin{array}{r} -7 = 6x-28 \\ +28 \quad \quad +28 \end{array}$$

$$\frac{21}{6} = \frac{6x}{6}$$

$$\frac{7}{2} = x$$

$$3.5 = x$$

$$\textcircled{7} \left(\sqrt[3]{x-4} \right)^3 = (5)^3$$

$$x-4 = 125$$

$$x = 129$$

$$\textcircled{8} \left(\sqrt{x^2 + 32} \right)^2 = (x + 2)^2$$

$$x^2 + 32 = (x + 2)(x + 2)$$

$$\cancel{x^2} + 32 = \cancel{x^2} + 4x + 4$$

$$32 = 4x + 4$$
$$\begin{array}{r} -4 \\ \hline \end{array}$$

$$28 = 4x$$
$$\textcircled{7 = x}$$

①

$$X^{\frac{1}{2}} = 5$$

$$\left(X^{\frac{1}{2}}\right)^2 = (5)^2$$

$$X^1 = 25$$

$$\left(\sqrt{x}\right)^2 = (5)^2$$

$$x = 25$$

$$\textcircled{12} \quad \underline{6(x+5)^{2/3}} = \underline{24}$$

$$\left[(x+5)^{2/3} \right]^{3/2} = (4)^{3/2}$$

$$x+5 = \sqrt[2]{4^3}$$

$$x+5=8$$

$$\textcircled{x=3}$$

$$\textcircled{13} \left(\sqrt{\sqrt{x+4}} \right)^2 = \left(\sqrt{x+2} \right)^2$$

$$\left(\sqrt{x+4} \right)^2 = \left(x+2 \right)^2$$

$$x+4 = (x+2)(x+2)$$

$$x+4 = x^2 + 4x + 4$$

$$\sqrt{\sqrt{-3+4}} = \sqrt{-3+2}$$

$$\sqrt{\sqrt{1}}$$

$$\sqrt{1}$$

$$\neq \sqrt{-1}$$

$$x = -3$$

extraneous

extraneous

$$\begin{array}{r} x+4 = x^2 + 4x + 4 \\ -x - 4 \quad \quad -x - 4 \end{array}$$

$$0 = x^2 + 3x$$

$$0 = x(x+3)$$

$$x=0, x+3=0$$

$$x=0, x=-3$$

extraneous

$$\textcircled{9} \quad X = \sqrt{2x-2} + 1$$

$$\begin{array}{c} -1 \\ \downarrow \\ (X-1)^2 = (\sqrt{2x-2})^2 \end{array}$$

$$(x-1)(x-1)$$

$$x^2 - 2x + 1 = 2x - 2$$

$$\begin{array}{l} x^2 - 2x + 1 = 2x - 2 \\ -2x + 1 \quad -2x + 2 \end{array}$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x-3=0, x-1=0$$

$$x=3, x=1$$

$$\textcircled{10} \quad (x-3)^2 = \left(\sqrt{-3x+19} \right)^2$$

$$(x-3)(x-3)$$

$$x^2 - 6x + 9 = -3x + 19$$

$$+3x - 19$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

extraneous

$$x = 5, x = -2$$

$$-5 \neq 5$$

$$x = \sqrt{-3x+19} + 3$$

$$(x-3)^2 = (\sqrt{-3x+19})^2$$

$$(x-3)(x-3) = -3x+19$$

$$\begin{array}{r} x^2 - 6x + 9 \\ + 3x - 19 \end{array} = \begin{array}{r} -3x + 19 \\ + 3x - 19 \end{array}$$

$$\begin{array}{l} x = -2 \\ -2 = 5 + 3 \end{array}$$

$$x^2 - 3x - 10 = 0$$

$$(x + 2)(x - 5) = 0$$

$$x + 2 = 0 \quad x - 5 = 0$$

$$x = -2, x = 5$$

extraneous

$$\textcircled{10} \quad \begin{array}{c} \overset{3}{=} \\ X = \sqrt{2x-2} + 1 \\ \underset{-1}{-1} \end{array}$$

$$\begin{array}{c} (X-1)^2 = (\sqrt{2x-2})^2 \\ \downarrow \\ (x-1)(x-1) \end{array}$$

$$\begin{array}{r} x^2 - 2x + 1 = 2x - 2 \\ -2x + 2 \end{array}$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x-3=0, x-1=0$$

$$\textcircled{x=3, x=1}$$



$$\textcircled{1} \quad (x-3)^2 = (\sqrt{-3x+19})^2$$

$$\begin{array}{r} (x-3)(x-3) \\ x^2 - 6x + 9 = -3x + 19 \\ +3x - 19 \end{array}$$

$$x^2 - 3x - 10 = 0$$

$$\begin{array}{r} (x-5)(x+2) = 0 \\ -5x + 2x \end{array}$$

$$\begin{array}{l} \textcircled{x=5} \\ \textcircled{x=-2} \end{array}$$

$$2 = 2 \checkmark$$

$$\underline{-5 = 5}$$

extraneous

$$\textcircled{10} \quad x = \sqrt{-3x+19} + 3$$

$$(x-3)^2 = (\sqrt{-3x+19})^2$$
$$(x-3)(x-3)$$

-27/8

$$x^2 - 6x + 9 = -3x + 19$$

$$\begin{array}{r} x^2 - 6x + 9 = -3x + 19 \\ + 5x - 19 \quad + 3x - 19 \end{array}$$

$$x^2 - 3x - 10 = 0$$

$$(x - 5)(x + 2) = 0$$

$$x = 5, x = -2$$

extraneous

$$\textcircled{12} \quad X^{\frac{1}{2}} = 3$$

$$\left(X^{\frac{1}{2}}\right)^2 = (3)^2$$

$$X = 9$$

$$\left(\sqrt{X}\right)^2 = (3)^2$$

$$X = 9$$

13

$$\underline{6(x+5)^{2/3} = 24}$$

$$\left[(x+5)^{2/3} \right]^{3/2} = 4^{3/2}$$

$$x+5 = \sqrt[2]{4^3} \quad x+5=8$$

$x=3$

14 $\left(\sqrt{\sqrt{x+49}}\right)^2 = \left(\sqrt{x+7}\right)^2$ $\sqrt{b} = \sqrt{-b}$

$$\left(\sqrt{x+49}\right)^2 = \left(x+7\right)^2$$

$$x+49 = (x+7)(x+7)$$

$$x+49 = x^2 + 14x + 49$$

$$\begin{array}{r} x+49 = x^2+14x+49 \\ -x-49 \quad \quad -x-49 \\ \hline \end{array}$$

$$0 = x^2 + 13x$$

extraneous

$$0 = x(x+13)$$

$$x=0, x+13=0$$

$$x=0, x=-13$$