6.5 Solving Radical Equations

① $\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

② $\left(\sqrt{x}\right)^2 = (3)^2$

$x = 9$

③ $\left(\sqrt{3x-2}\right)^2 = (7)^2$

$3x-2 = 49$

$+2 \quad +2$

$\frac{3x}{3} = \frac{51}{3}$

$x = 17$

(4) $3\sqrt{5-x}+2=8$

$3y+2=8$

$-2 \quad -2$

$\frac{3y}{3}=\frac{6}{3}$

$y=2$

$(\sqrt{5-x})^2=(2)^2$

$5-x=4$

$-5 \quad -5$

$-x=-1$

$x=1$

(5) $5\sqrt{2x-7} - 3 = 42$

$+3 \quad +3$

$\frac{5\sqrt{2x-7}}{5} = \frac{45}{5}$

$\left(\sqrt{2x-7}\right)^2 = (9)^2$

$$2x - 7 = 81$$

$$+7 \quad +7$$

$$\frac{2x}{2} = \frac{88}{2}$$

$$x = 44$$

⑥ $\left(\sqrt{5x-7}\right)^2 = \left(\sqrt{11x-28}\right)^2$

$5x-7 = 11x-28$

$-5x \qquad -5x$

$-7 = 6x-28$

$+28 \qquad +28$

$$\frac{21}{6} = \frac{6x}{6}$$

$$\frac{7}{2} = x$$

$$3.5 = x$$

7 $\left(\sqrt[3]{x-4}\right)^3 = (5)^3$

$x-4 = 125$

$x = 129$

⑧ $\left(\sqrt{x^2+32}\right)^2 = (x+2)^2$

$x^2+32 = (x+2)(x+2)$

$x^2+32 = x^2+4x+4$

$-x^2 \quad\quad -x^2$

$32 = 4x+4$

$-4 \quad\quad -4$

$28 = 4x$

$7 = x$

(11) $x^{1/2} = 5$

$\left(x^{1/2}\right)^2 = (5)^2$

$x^1 = 25$

$\left(\sqrt{x}\right)^2 = (5)^2$

$x = 25$

(12) $\frac{6(x+5)^{2/3}}{6} = \frac{24}{6}$

$$\left[(x+5)^{2/3}\right]^{3/2} = (4)^{3/2}$$

$$x+5 = \sqrt[2]{4^3}$$

$x+5=8$

$x=3$

13 $\left(\sqrt{\sqrt{x+4}}\right)^2 = \left(\sqrt{x+2}\right)^2$

$\left(\sqrt{x+4}\right)^2 = (x+2)^2$

$x+4 = (x+2)(x+2)$

$2x$ $2x$

$x+4 = x^2+4x+4$

$$\sqrt{\sqrt{-3+4}} = \sqrt{-3+2}$$

$$\sqrt{\sqrt{1}}$$

$$\sqrt{1}$$

$$1 \neq \sqrt{-1}$$

$x = -3$ extraneous

extraneous

$$x+4 = x^2+4x+4$$

$$-x-4 \qquad -x-4$$

$$0 = x^2+3x$$

$$0 = x(x+3)$$

$$x=0 \;,\; x+3=0$$

extraneous

$$\boxed{x=0,\; x=-3}$$

9 $x = \sqrt{2x-2} + 1$

$-1 \quad -1$

$(x-1)^2 = (\sqrt{2x-2})^2$

$(x-1)(x-1)$

$x^2 - 2x + 1 = 2x - 2$

$x^2 - 2x + 1 = 2x - 2$

$-2x+2 \quad -2x+2$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x-3=0 \quad , x-1=0$

$x = 3, x = 1$

10 $(x-3)^2 = \left(\sqrt{-3x+19}\right)^2$

$(x-3)(x-3)$

$x^2-6x+9 = -3x+19$

$+3x-19 \quad +3x-19$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5, x=-2$

extraneous

$-5 \neq 5$

$$x = \sqrt{-3x+19} + 3$$

$$(x-3)^2 = \left(\sqrt{-3x+19}\right)^2$$

$$(x-3)(x-3) = -3x+19$$

$$x^2 - 6x + 9 = -3x + 19$$

$$+3x \quad -19 \qquad +3x \quad -19$$

$x = -2$

$-2 = 5 + 3$

$$x^2 - 3x - 10 = 0$$

$$(x+2)(x-5) = 0$$

$x+2=0 \quad x-5=0$

$x = -2, \; x = 5$

extraneous

(10) $x = \sqrt{2x-2} + 1$

$-1 \qquad -1$

$(x-1)^2 = (\sqrt{2x-2})^2$

$(x-1)(x-1)$

$x^2 - 2x + 1 = 2x - 2$

$-2x + 2 \qquad -2x + 2$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x - 3 = 0, \; x - 1 = 0$

$x = 3, \; x = 1$

✓

11 $(x-3)^2 = (\sqrt{-3x+19})^2$

$(x-3)(x-3)$

$x^2 - 6x + 9 = -3x + 19$

$+3x - 19 \quad +3x - 19$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$-5x + 2x$

$x = 5$ $x = -2$

$2 = 2$ ✓

$-5 = 5$

extraneous

(10) $x = \sqrt{-3x+19} + 3$

$-3 \qquad\qquad -3$

$(x-3)^2 = \left(\sqrt{-3x+19}\right)^2$

$-2 \neq 8$

$(x-3)(x-3)$

$x^2 - 6x + 9 = -3x + 19$

$$x^2 - 6x + 9 = -3x + 19$$

$$+3x - 19 \qquad +3x - 19$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$x = 5, x = -2$ → extraneous

(12) $x^{1/2} = 3$

$\left(x^{1/2}\right)^2 = (3)^2$

$x^1 = 9$

$\left(\sqrt[2]{x^1}\right)^2 = (3)^2$

$x = 9$

(13)

$$\frac{6(x+5)^{2/3}}{6} = \frac{24}{6}$$

$$\left[(x+5)^{2/3}\right]^{3/2} = 4^{3/2}$$

$$x+5 = \sqrt{4^3} \qquad x+5=8$$

$$x=3$$

14 $\left(\sqrt{\sqrt{x+49}}\right)^2 = \left(\sqrt{x+7}\right)^2$ $\sqrt{6} = \sqrt{-6}$

$\left(\sqrt{x+49}\right)^2 = \left(x+7\right)^2$

$x+49 = (x+7)(x+7)$

$x+49 = x^2+14x+49$

$x + 49 = x^2 + 14x + 49$

$-x \quad -49 \qquad -x \quad -49$

$0 = x^2 + 13x$

$0 = x(x+13)$

$x = 0 \quad , x + 13 = 0$

$x = 0, x = -13$

extraneous