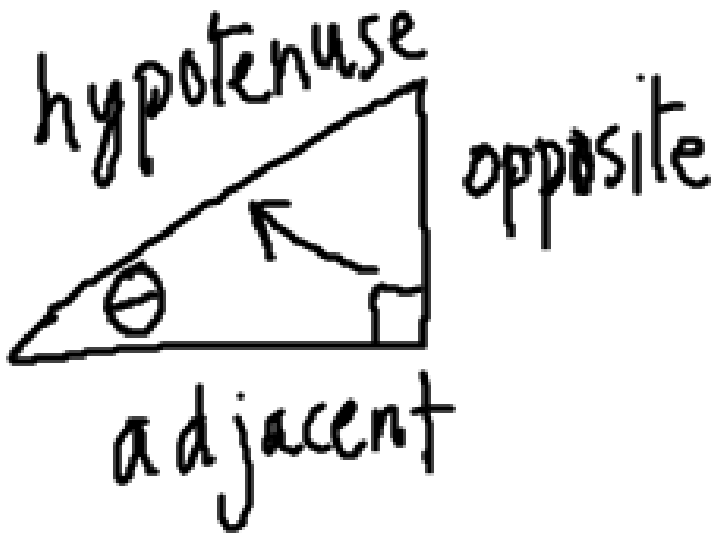
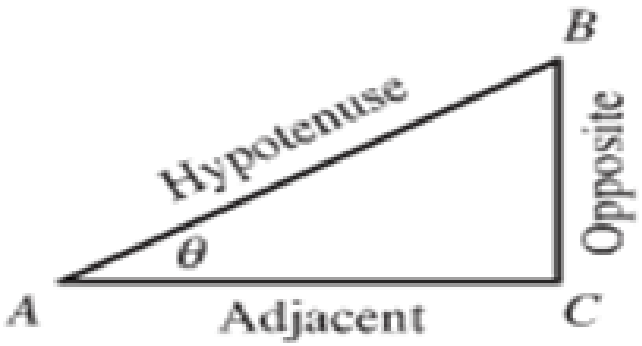
Right $\triangle$ Trigonometry





SOH-CAH-TOA

FIGURE 4.8 The triangle referenced in our definition of the trigonometric functions.

DEFINITION Trigonometric Functions

Let θ be an acute angle in the right ΔABC (Figure 4.8). Then

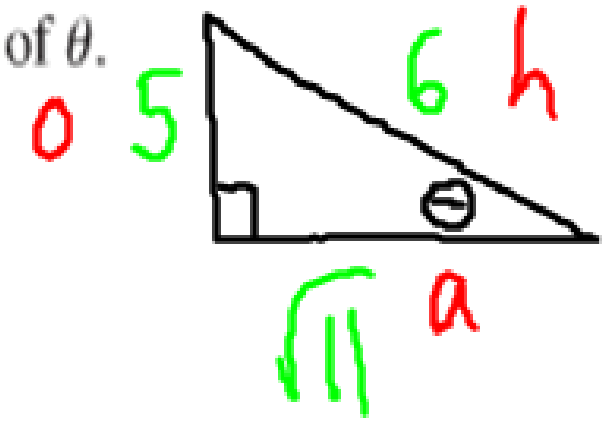
$$\textbf{sine } (\theta) = \sin\theta = \frac{opp}{hyp} \qquad \textbf{cosecant } (\theta) = \csc\theta = \frac{hyp}{opp}$$

$$\textbf{cosine } (\theta) = \cos\theta = \frac{adj}{hyp} \qquad \textbf{secant } (\theta) = \sec\theta = \frac{hyp}{adj}$$

$$\textbf{tangent } (\theta) = \tan\theta = \frac{opp}{adj} \qquad \textbf{cotangent } (\theta) = \cot\theta = \frac{adj}{opp}$$

EXAMPLE 3 Using One Trigonometric Ratio to Find Them All

Let θ be an acute angle such that $\sin\theta = 5/6$. Evaluate the other five trigonometric functions of θ.



$$\sin\theta = \frac{5}{6} \quad \frac{o}{h}$$

$$a^2 + 5^2 = 6^2$$

$$a^2 = 11$$

$$a = \sqrt{11}$$

$$\sin\theta = \frac{5}{6}$$

$$\csc\theta = \frac{6}{5}$$

$$\cos\theta = \frac{\sqrt{11}}{6}$$

$$\sec\theta = \frac{6}{\sqrt{11}}$$

$$\tan\theta = \frac{5}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\cot\theta = \frac{\sqrt{11}}{5}$$

EXAMPLE 5 Solving a Right Triangle

A right triangle with a hypotenuse of 8 includes a 37° angle (Figure 4.17). Find the measures of the other two angles and the lengths of the other two sides.

SOLUTION Since it is a right triangle, one of the other angles is 90°. That leaves $180° - 90° - 37° = 53°$ for the third angle.

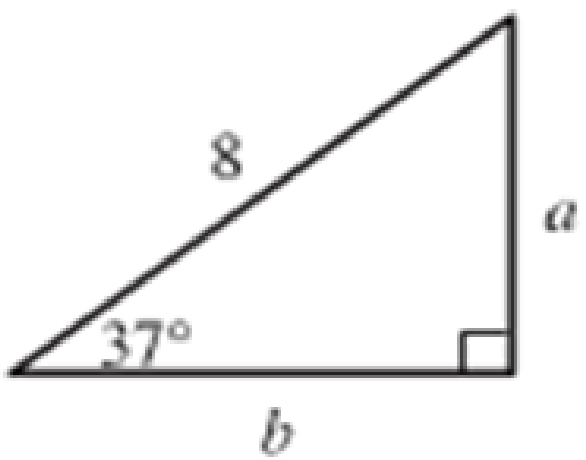
Referring to the labels in Figure 4.17, we have

$$\sin 37° = \frac{a}{8} \qquad \cos 37° = \frac{b}{8}$$

$$a = 8 \sin 37° \qquad b = 8 \cos 37°$$

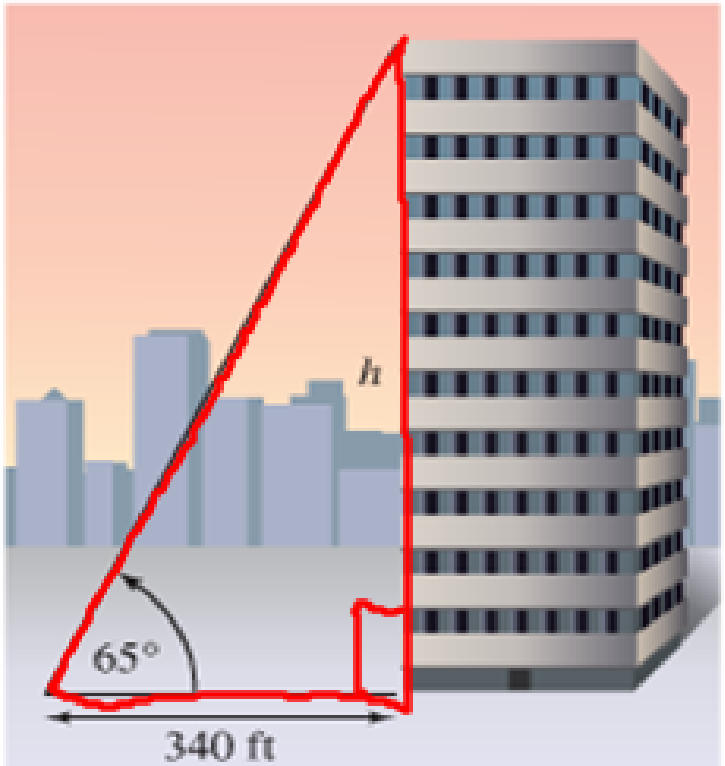
$$a \approx 4.81 \qquad b \approx 6.39$$

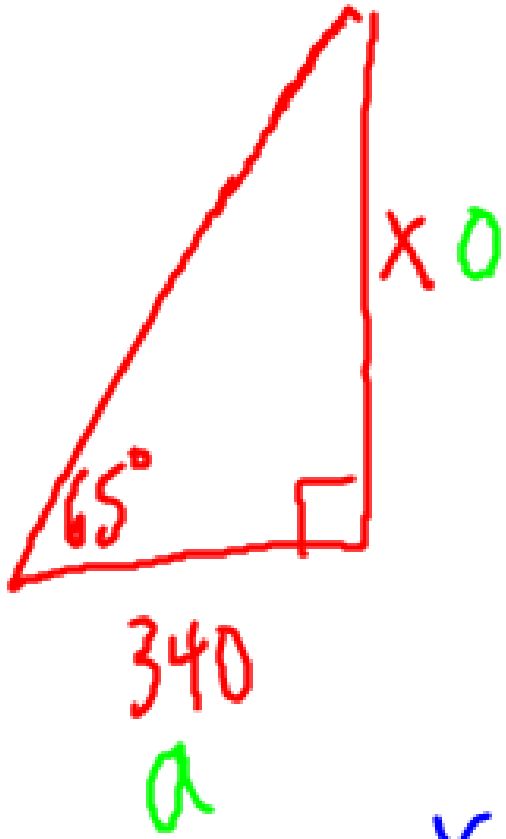
Now try Exercise 55



EXAMPLE 6 Finding the Height of a Building

From a point 340 feet away from the base of the Peachtree Center Plaza in Atlanta, Georgia, the angle of elevation to the top of the building is 65°. (See Figure 4.18.) Find the height h of the building.





$$\frac{\tan 65^\circ}{1} = \frac{x}{340}$$

$$x = 340 \tan 65^\circ$$

$$x \approx 729.13 \text{ ft.}$$

3

$$\frac{\cos 40^\circ}{1} = \frac{7}{x}$$

$$\frac{x\cos 40}{\cos 40} = \frac{7}{\cos 40}$$

$$x = \frac{7}{\cos 40}$$

$$x \approx 9.14$$