3.6 Chain Rule

$y = f \circ g = f(g(x))$

$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

$u = g(x)$

$y = f(u)$

$y' = \frac{dy}{du} \cdot \frac{du}{dx}$

① $y = (2x+3)^2$

$g(x) = 2x+3 = u$

$f(x) = u^2$

$y = u^2$

$y' = 2u \cdot 2$

$y' = 2(2x+3) \cdot 2$

$y' = 8x+12$

$y' = 2(2x+3)^1 \cdot 2$

② $y = (x^3 + x - 1)^5$ $u = x^3 + x - 1$

$y = u^5$

$y' = \frac{dy}{du} \cdot \frac{du}{dx}$

$y' = 5u^4(3x^2 + 1)$

$y' = 5(x^3 + x - 1)^4(3x^2 + 1)$

(3) $y = \sqrt{x^2+4}$ $\quad u = x^2+4$

$y = u^{1/2}$

$y' = \frac{dy}{du} \cdot \frac{du}{dx}$

$y' = \frac{1}{2} u^{-1/2} \cdot (2x)$

$y' = \frac{1 \cdot 2x}{2u^{1/2}}$

$y' = \frac{x}{\sqrt{x^2+4}}$

(4) $y = (2x+3)^{2/3}$ $\quad u = 2x+3$

$y = u^{2/3}$

$y' = \frac{2}{3} u^{-1/3} \cdot 2$

$y' = \frac{4}{3\sqrt[3]{2x+3}}$

⑤ $y = \sin 2x$ $\quad u = 2x$

$y = \sin u$

$y' = \frac{dy}{du} \cdot \frac{du}{dx}$

$y' = \cos u \cdot 2$

$y' = 2 \cos 2x$

(6) $y = \sec(x^2)$ $\quad u = x^2$

$y = \sec u$

$y' = \sec u \tan y \cdot 2x$

$y' = 2x \sec x^2 \tan x^2$