1.5 Inverses

Inverses undo one another like $+/-$, $\cdot/\div$, $(\ )^2\ \sqrt{\ }$, etc.

To find inverses algebraically

1. Replace $f(x)$ with y.
2. Switch x and y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$

"inverse of $f(x)$"

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① Find $f^{-1}(x)$ if $f(x) = 5x+2$

$y = 5x+2$

$x = 5y+2$

$x-2 = 5y$

$\frac{x-2}{5} = y$

$\frac{x-2}{5} = f^{-1}(x)$

② Find $f^{-1}(x)$ if $f(x) = \sqrt[3]{x-7}$

$y = \sqrt[3]{x-7}$

$x = \sqrt[3]{y-7}$

$x^3 = y-7$

$x^3+7 = y$

$x^3+7 = f^{-1}(x)$

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③ Find $f^{-1}(x)$ if $f(x) = \sqrt{x} + 5$

$y = \sqrt{x} + 5$

$x = \sqrt{y} + 5$

$x-5 = \sqrt{y}$

$(x-5)^2 = y$

$(x-5)^2 = f^{-1}(x)$

If functions are inverses, then $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

④ Show f and g are inverses if $f(x) = x^3+5$; $g(x) = \sqrt[3]{x-5}$

$f(g(x)) = (\sqrt[3]{x-5})^3 + 5 = x-5+5 = x$

$g(f(x)) = \sqrt[3]{(x^3+5)-5} = \sqrt[3]{x^3} = x$

⑤ Show f and g are inverses if $f(x) = 7x-2$; $g(x) = \frac{x+2}{7}$

$f(g(x)) = 7\left(\frac{x+2}{7}\right) - 2 = x+2-2 = x$

$g(f(x)) = \frac{(7x-2)+2}{7} = \frac{7x}{7} = x$

SECTION 1.5 EXERCISES

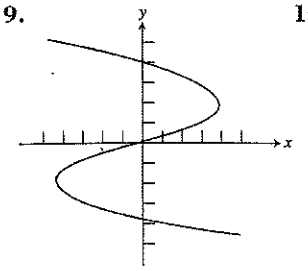
In Exercises 1–4, find the (x, y) pair for the value of the parameter.

1. $x = 3t$ and $y = t^2 + 5$ for $t = 2$
2. $x = 5t - 7$ and $y = 17 - 3t$ for $t = -2$
3. $x = t^3 - 4t$ and $y = \sqrt{t + 1}$ for $t = 3$
4. $x = |t + 3|$ and $y = 1/t$ for $t = -8$

In Exercises 5–8, complete the following. **(a)** Find the points determined by $t = -3, -2, -1, 0, 1, 2$, and 3. **(b)** Find a direct algebraic relationship between x and y and determine whether the parametric equations determine y as a function of x. **(c)** Graph the relationship in the xy-plane.

5. $x = 2t$ and $y = 3t - 1$
6. $x = t + 1$ and $y = t^2 - 2t$
7. $x = t^2$ and $y = t - 2$
8. $x = \sqrt{t}$ and $y = 2t - 5$

In Exercises 9–12, the graph of a relation is shown. **(a)** Is the relation a function? **(b)** Does the relation have an inverse that is a function?

9.


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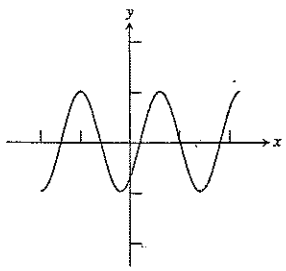

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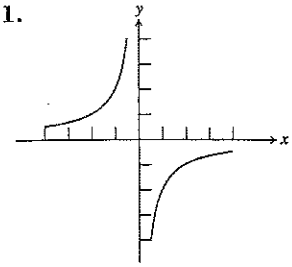

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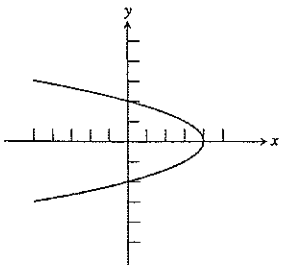

In Exercises 13–22, find a formula for $f^{-1}(x)$. Give the domain of f^{-1}, including any restrictions "inherited" from f.

13. $f(x) = 3x - 6$
14. $f(x) = 2x + 5$
15. $f(x) = \dfrac{2x - 3}{x + 1}$
16. $f(x) = \dfrac{x + 3}{x - 2}$
17. $f(x) = \sqrt{x - 3}$
18. $f(x) = \sqrt{x + 2}$
19. $f(x) = x^3$
20. $f(x) = \sqrt{x^3 + 5}$
21. $f(x) = \sqrt[3]{x + 5}$
22. $f(x) = \sqrt[3]{x - 2}$

In Exercises 23–26, determine whether the function is one-to-one. If it is one-to-one, sketch the graph of the inverse.

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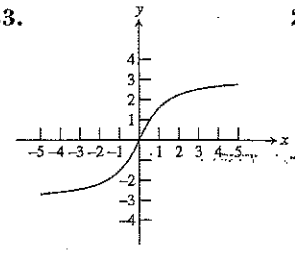

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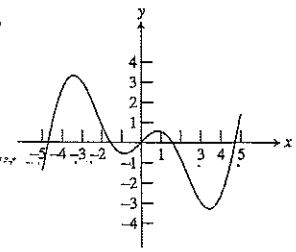

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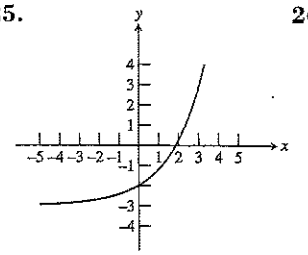

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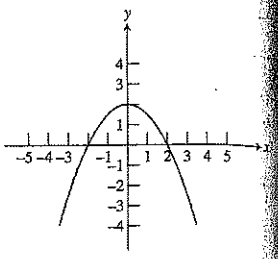


In Exercises 27–32, confirm that f and g are inverses by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

27. $f(x) = 3x - 2$ and $g(x) = \dfrac{x + 2}{3}$
28. $f(x) = \dfrac{x + 3}{4}$ and $g(x) = 4x - 3$
29. $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x - 1}$
30. $f(x) = \dfrac{7}{x}$ and $g(x) = \dfrac{7}{x}$
31. $f(x) = \dfrac{x + 1}{x}$ and $g(x) = \dfrac{1}{x - 1}$
32. $f(x) = \dfrac{x + 3}{x - 2}$ and $g(x) = \dfrac{2x + 3}{x - 1}$

33. **Currency Conversion** In May of 2002 the exchange rate for converting U.S. dollars (x) to euros (y) was $y = 1.08x$.
   (a) How many euros could you get for \$100 U.S.?
   (b) What is the inverse function, and what conversion does it represent?
   (c) In the spring of 2002, a tourist had an elegant lunch in Provence, France, ordering from a "fixed price" 48-euro menu. How much was that in U.S. dollars?

43. **Multiple Choice** Which function is the *inverse* of the function $f(x) = 3x - 2$?

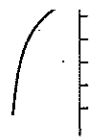
   (A) $g(x) = \dfrac{x}{3} + 2$
   (B) $g(x) = 2 - 3x$
   (C) $g(x) = \dfrac{x + 2}{3}$
   (D) $g(x) = \dfrac{x - 3}{2}$
   (E) $g(x) = \dfrac{x - 2}{3}$

38. Which basic function can be defined parametrically as follows?

$$x = 8t^3 \text{ and } y = (2t)^3 \text{ for } -\infty < t < \infty$$

9. (a) No (b) Yes **11.** (a) Yes (b) Yes **13.** $f^{-1}(x) = \frac{1}{3}x + 2, (-\infty, \infty)$ **15.** $f^{-1}(x) = \frac{x+3}{2-x}, (-\infty, 2) \cup (2, \infty)$

17. $f^{-1}(x) = x^2 + 3, x \geq 0$ **19.** $f^{-1}(x) = \sqrt[3]{x}, (-\infty, \infty)$ **21.** $f^{-1}(x) = x^3 - 5, (-\infty, \infty)$



27. $f(g(x)) = 3\left[\frac{1}{3}(x+2)\right] - 2 = x + 2 - 2 = x; g(f(x)) = \frac{1}{3}[(3x-2)+2] = \frac{1}{3}(3x) = x$

29. $f(g(x)) = [(x-1)^{1/3}]^3 + 1 = (x-1)^1 + 1 = x - 1 + 1 = x; g(f(x)) = [(x^3+1) - 1]^{1/3} = (x^3)^{1/3} = x^1 = x$

31. $f(g(x)) = \dfrac{\frac{1}{x-1} + 1}{\frac{1}{x-1}} = (x-1)\left(\frac{1}{x-1} + 1\right) = 1 + x - 1 = x; g(f(x)) = \dfrac{1}{\frac{x+1}{x} - 1} = \left(\dfrac{1}{\frac{x+1}{x} - 1}\right) \cdot \frac{x}{x} = \dfrac{x}{x+1-x} = \frac{x}{1} = x$

33. (a) 108 euros (b) $y = \frac{25}{27}x$. This converts euros (x) to dollars (y). (c) \$44.44 **35.** $y = e^x$ and $y = \ln x$ are inverses. If we restrict the domain of the function $y = x^2$ to the interval $[0, \infty)$, then the restricted function and $y = \sqrt{x}$ are inverses. **37.** $y = |x|$

39. True. All the ordered pairs swap domain and range values. **41.** E **43.** C **45.** Answers may vary. (a) If the graph of f is unbroken, its reflection in the line $y = x$ will be also. (b) Both f and its inverse must be one-to-one in order to be inverse functions. (c) Since f is odd, $(-x, -y)$ is on the graph whenever (x, y) is. This implies that $(-y, -x)$ is on the graph of f^{-1} whenever (x, y) is. That implies that f^{-1} is odd. (d) Let $y = f(x)$. Since the ratio of Δy to Δx is positive, the ratio of Δx to Δy is positive. Any ratio of Δy to Δx on the graph of f^{-1} is the same as some ratio of Δx to Δy on the graph of f, hence positive. This implies that f^{-1} is increasing.

47. (a) $y = 0.75x + 31$ (b) $y = \frac{4}{3}(x - 31)$. It converts scaled scores to raw scores. **49.** (a) No (b) No (c) 45°; yes

51. When $k = 1$, the scaling function is linear. Opinions will vary as to which is the best value of k.

SECTION 1.6

Exploration 1

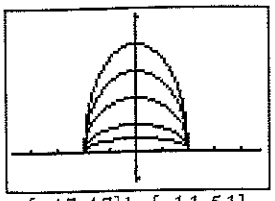
1. They raise or lower the parabola along the y-axis. **3.** Yes

Exploration 2

1. Graph C. Points with positive y-coordinates remain unchanged, while points with negative y-coordinates are reflected across the x-axis.
3. Graph F. The graph will be a reflection across the y-axis of graph C.

Exploration 3

1.



[−4.7, 4.7] by [−1.1, 5.1]

The 1.5 and the 2 stretch the graph vertically; the 0.5 and the 0.25 shrink the graph vertically.

Quick Review 1.6

1. $(x+1)^2$ **3.** $(x+6)^2$ **5.** $(x - 5/2)^2$ **7.** $x^2 - x + 2$ **9.** $x^3 - 6x + 5$

Exercises 1.6

1. Vertical translation down 3 units **3.** Horizontal translation left 4 units **5.** Horizontal translation to the right 100 units
7. Horizontal translation to the right 1 unit, and Vertical translation up 3 units **9.** Reflection across x-axis **11.** Reflection across y-axis
13. Vertically stretch by 2 **15.** Horizontally stretch by $\frac{1}{0.2} = 5$, or vertically shrink by $0.2^3 = 0.008$ **17.** Translate right 6 units to get g